Transient 3-D Model for Lifting, Transporting, and Depositing Solid Material

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Introduction

Sediment scour and deposition is an important consideration for the design of dams, reservoirs, piping systems, settling ponds, and flood control devices, to name but a few applications. To more accurately compute the effect of erosion and sedimentation in a transient three-dimensional system is challenging. Empirical methods based on depth-averaged models are not capable of determining local scour and sedimentation conditions. This paper presents a computational method based on the fundamental laws of mass, momentum and energy conservation. Finite-difference approximations are used to enforce conservation equations. The flow model uses a fixed-grid of cells which are all rectangular prisms; the fluid moves through the grid and free surfaces are tracked with the Volume-of-Fluid (VOF) technique. Sediment is treated as a continuous phase, a further distinguished by packed a suspended phases. The two components of scour, drifting and lifting, are superimposed on the advection of the sediment with the fluid. The effects of particle interaction and the angle of repose of the sediment are included in the model. Regions where the sediment reaches the packing concentration are frozen, mimicking a solid material.

Basic model approach

The sediment particles are presumed to be small relative to the scale of the flow. In regions where the sediment is suspended in the fluid, the sediment is tracked as *suspended sediment*, which advects with the fluid flow. The advection of sediment is computed from

$$V_F \frac{\partial C_s}{\partial t} + \nabla \cdot \left(\mathbf{uA} C_s \right) = Diffusion\, terms$$

where V_F is the volume fraction of liquid present in the computational cell, A is the area fractions of each of the three directions open to fluid flow, $\mathbf{u}$ is the local fluid velocity and C_s is the local concentration of sediment. The advection equation is discretized by a first-order accurate finite difference scheme. The fluid density is computed as the volume-weighted average of the fluid and sediment, and this density is used in the conservation of momentum equations for the fluid flow. The Renormalization-Group Model for formation and dissipation of turbulent energy is incorporated into the flow model [1,2]; the presence of sediment does not affect the formation or dissipation of turbulent energy, but the turbulent energy can affect the dissipation of sediment.

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Additionally, sediment particles drift relative to the fluid due to the density difference between the sediment particles and the fluid. This drift velocity is computed as the balance between buoyant forces and the fluid's drag on the particle. The drag is computed with the assumption of Stokes flow around spherical sediment particles and the buoyant forces are based on the mechanical potential gradient to include effects of centripetal acceleration in the flow around bends:

$$\mathbf{u}_{drift} = \frac{d^2\left(\rho_s - \rho_f\right)}{18\mu} \underbrace{\frac{\nabla P}{\bar{\rho}}}_{\substack{\text{Mechanical potential} \\ \text{gradient}}} .$$

Here $\mathbf{u}_{drift}$ is the velocity of the fluid particle relative to the fluid, d is the average particle diameter, μ is the local fluid viscosity, ρ_f and ρ_s are the fluid and microscopic particle densities, respectively, ∇P is the local pressure gradient and $\bar{\rho}$ is the local macroscopic density.

In regions where the sediment drifts and accumulates (e.g. on the inside curve of a meandering stream), the suspended sediment becomes *packed sediment*, which does not advect with the fluid; fluid flow ceases in these regions. However, packed sediment can be re-suspended into the fluid if the shear stress at the packed bed – fluid interface is sufficient to pick up the sediment particles. In turbulent flow, the shear stress is enhanced by the local turbulent kinetic energy. The lift velocity is computed as

$$u_{lift} = \alpha\sqrt{\frac{\tau - \tau_{crit}}{\bar{\rho}}} .$$

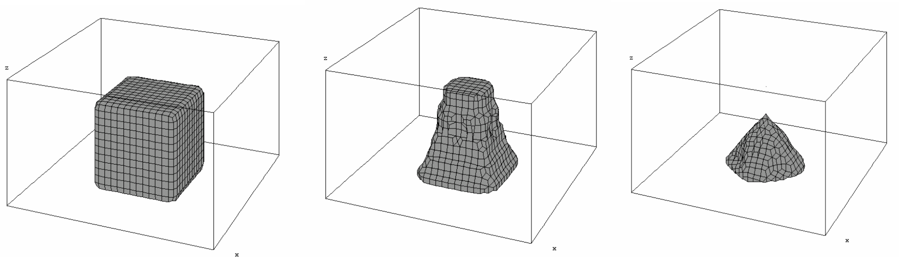
Here α is the scour parameter, τ is the local shear stress at the packed bed interface, and τ_{crit} is the critical shear stress. In areas on the packed bed interface where τ is less than τ_{crit}, no scour lift occurs. The direction of u_{lift} is always away from the packed bed interface. The rationale behind this empirical model is that $\sqrt{\tau / \rho}$ is the *shear velocity* [3], and is a measure of the velocity within the boundary layer. The scour parameter α is the probability that a sediment particle on the packed bed interface is lifted away; its value is typically close to 1. Also, α can be increased to model qualitative behavior on an accelerated time frame. τ_{crit} is calculated from the critical Shields Parameter, θ_{crit}:

$$\tau_{crit} = \theta_{crit}\, g\left(\rho_s - \rho_f\right)d .$$

Here g is the magnitude of the gravity vector, $\mathbf{g}$. θ_{crit} is found in the literature for various materials.

Effect of interface slope

τ_{crit} applies for flat packed-bed interfaces. Where the interface is sloped, the amount of traction needed to initiate motion is reduced. This reduction can be defined with the angle of repose [3] of the sediment; this is the angle beyond which the slope is unstable. The reduced critical shear stress is:



(a) 0 s (b) 9 s (c) 30 s

Figure 1. A 25 cm cube of sand collapses to an equilibrium slope of 45°, the angle of repose specified. The final volume of the packed sediment is much smaller than the initial volume because the eroded sediment has left the domain.

$$(\tau_{crit})_{Slope} = (\tau_{crit})_{No\ Slope}\sqrt{1-\frac{\sin^2\varphi}{\sin^2\zeta}}$$

Here φ is the local slope of the packed bed of sediment and ζ is the angle of repose. In regions where the local slope exceeds the angle of repose, particles flow even in static fluid.

Figure 1 shows the effect of the angle of repose. Suppose a block of packed sediment with an angle of repose of 45° is initialized as a cubic shape as shown in Fig. 1a (the block is a 25 cm cube) in static fluid. The sediment along the sides of the cube does not have the cohesive strength to maintain that slope, so it spontaneously slides down and is advected with the fluid out of the domain. This continues until all parts of the interface do not exceed the angle of repose. Fig. 1b shows the intermediate case after 15 seconds, and Fig. 1c shows the equilibrium sediment ‘pile’ after 30 seconds. The sediment in this example has an average particle diameter of 2 mm. Figure 2 shows a two-dimensional cross-section of the same system; note the equilibrium slope. In both cases the critical Shields parameter is set to 0.04.

Validation

The model was tested with experimental flume data; a submerged horizontal jet, 2cm wide and with a pressure head of 11.8 cm H2O, is introduced along the surface of a solid apron, 66 cm long. Beyond the apron, the bed of sand extends

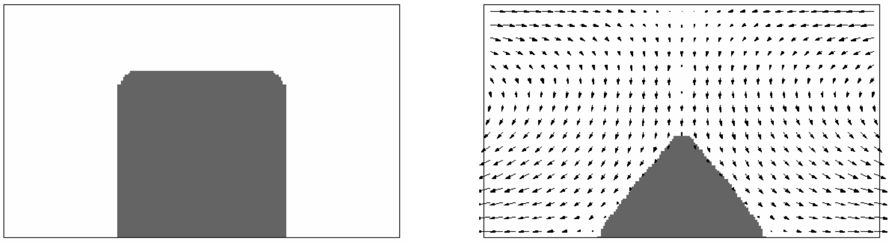


Figure 2. Two-dimensional slice of ‘sand pile’ shown in Fig. 1. Note that the equilibrium slope on the right is about 45°.

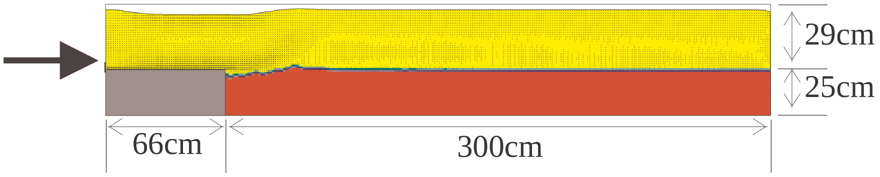
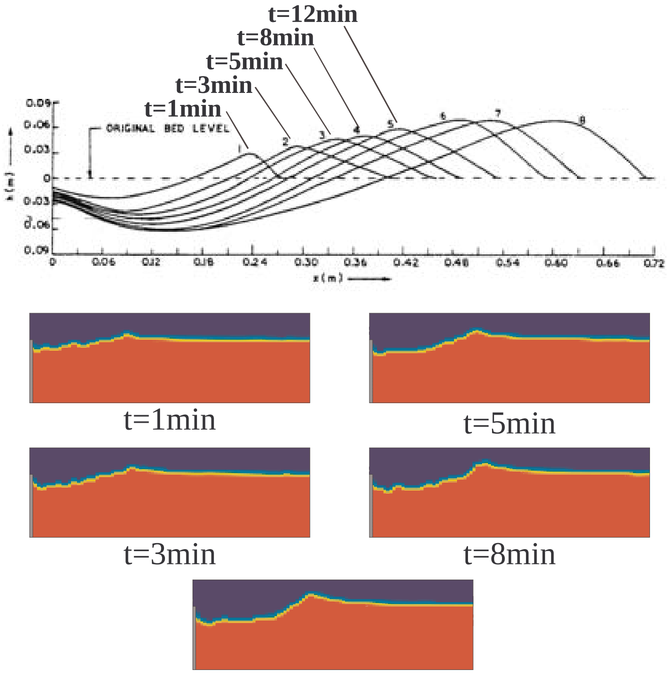


Figure 3. Profile of experimental flume setup for submerged horizontal jet problem [4].

for 3m lengthwise, 25 cm deep and 60 cm wide. All measurements were made in the centerline of the flume, so the profile is taken to be two-dimensional. The average particle diameter of the sand is 0.76 mm and its specific gravity is 2.65. The Shields parameter is taken to be 0.04 in this system [4]. As mentioned previously, the Renormalization-Group model of turbulence formation and dissipation was used, and the suspended sediment was presumed to dissipate just as does the turbulent energy. A first order finite difference approximation of the sediment transport equation was used to predict advection of the suspended sediment. Fig. 3 shows a two-dimensional slice of the flume setup. Fig. 4 shows the packed sediment bed profiles after 1, 3, 5, 8 and 12 minutes of scour, along with the experimentally measured profiles [4]. Fig. 5 plots the comparison of maximum trough depth and maximum ridge height as a function of time for both the experimental measurements and the results predicted by this model.



t=1min
t=5min
t=3min
t=8min
t=12min

Figure 4. Top plot shows the time evolution of the packed bed profile based on experiment [4]. The bottom plots show the predicted profiles for the packed sediment bed.

The predicted packed sand bed profile compares well in the time evolution to that of the experiments. The trough and ridge both move downstream with time while the trough deepens and the ridge grows. Also, the ridge peak becomes less sharp as time advances.

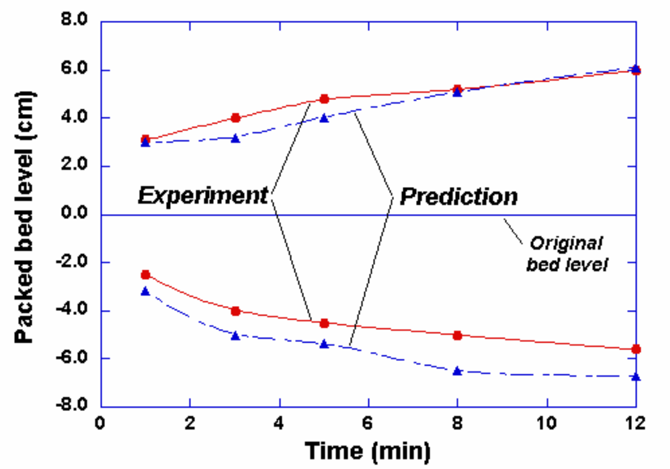


Figure 5. Plot of time evolution of packed sediment trough depths (bottom curves) and ridge peak heights (upper curves) for experiment [4] and prediction (this model).

Dam failure analysis

Figure 6 shows the time evolution of an earthen dam that has begun to leak; in this case the leak is at the top of the dam. The dam material has an average particle size of 0.14 mm, an angle of repose of 45°, and the critical Shields parameter is 0.04. The dam is is 200 m high from its base, 50 m wide at the top and its slopes are 40°. The computation starts with an eroded gully; the water flowing through the gully erodes the gully deeper and wider, eventually causing failure. The erosion begins slowly because the shear stress and turbulence in the gully are both small. At later times, the flow of water increases as the depth and width of the gully rise, causing the flow rate, shear stress and turbulent mixing to rise; erosion accelerates. Finally, complete failure occurs after 3 minutes. Note that in regions where the slope of the dam material exceeds 45°, the material crumbles away, as expected. Also note that the erosion of the dam is not symmetric, due to the turbulent flow; slight differences in the turbulence and shear stress at the packed sediment interface cause differences in geometry, which causes greater differences in the flow. This feedback effect creates the asymmetric behavior visible in Figure 6.

Summary

A new scour model for computing the erosion and deposition of sediment has been presented. Although scour is a chaotic process, this model, which is based on empirical relationships of scour lift along with an accurate computation of the fluid flow, shows that quantitative comparisons can be made. This model has been incorporated into ***FLOW-3D®***, a commercial CFD code [5], and is illustrated here through application to a variety of sample problems.

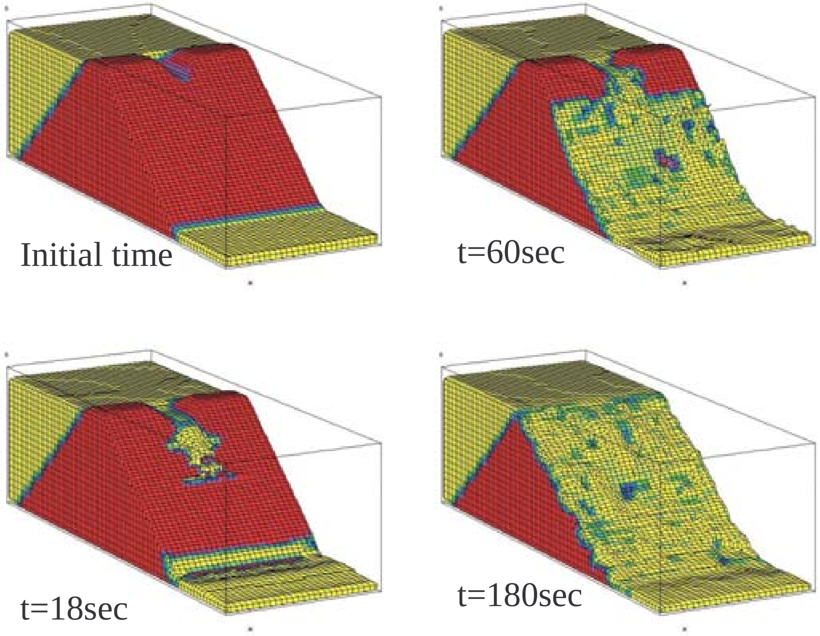


Figure 6. Three-dimensional views of time progression of dam break problem. The dark shaded region is the packed sediment (the dam) and the light shaded region is water or water/sediment mixture. In this case, a rupture has presumed to have occurred at the initial time, and failure occurs after only 3 min.

Acknowledgements

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