Three-Dimensional Calculation of Bubble Growth and Drop Ejection in a Bubble Jet Printer

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The three-dimensional Navier-Stokes equation for the motion of ink both inside and outside the nozzle of a bubble jet printer is numerically solved, for the first time, to predict the bubble behavior and the drop ejection. The results of calculation for three types of ink agreed well with experimental data. The effect of initial bubble pressure, viscosity and surface tension on the volume and the velocity of the drop is numerically investigated. The three-dimensional calculation is very useful to the design of bubble jet printers because it saves a lot of time and cost to make and evaluate prototypes.

Introduction

The bubble jet is a new type of drop-on-demand ink jet printing technology, in which vapor bubbles, generated by a rapid heating of ink in nozzles, act as pressure generators (Hara and Endo, 1982). Ink drops are ejected from the outlet of nozzles by the action of high pressure due to boiling under extremely high superheat. A great merit of the bubble jet is that small pitched multiple nozzle arrangements, which are difficult to obtain using piezoelectric transducers, can be achieved by using thin-film resistors as the heating means.

In designing bubble jet printers, it is necessary to know the relation between design parameters (such as ink property and head structure) and the performance characteristics. Numerical simulation of the drop ejection is desired for this purpose, since building prototypes and evaluating their performance are expensive and time-consuming tasks.

For piezoelectric ink jets, Kawamura et al. (1982) and Fromm (1984) presented axisymmetric calculations to predict the drop behavior outside the nozzle. In these calculations, time dependent history of pressure or velocity field is assumed at the outlet of the nozzle. Such method is not satisfactory for bubble jets because the pressure history at the nozzle outlet is not known a priori but should be determined by the behavior of the bubble in the nozzle.

For bubble jets, Allen et al. (1985) presented an axisymmetric calculation of bubble growth and drop formation. But the bubble dynamics law, which is very important for the calculation, is not shown. Further, an axisymmetric calculation is not satisfactory because the three-dimensional effect of the nozzle shape cannot be neglected in many practical situations.

In this paper, a three-dimensional finite difference algorithm based on the VOF method (Hirt and Nichols, 1981 and Flow Science, 1988) is used to predict the bubble behavior and the drop ejection in a bubble jet printer. The results of calculation for three types of ink are compared with experimental data. Further, the effect of ink property on the drop size and velocity is numerically investigated.

Method of Calculation

Fluid Dynamic Equations. The motion of ink is described by the three-dimensional Navier-Stokes equation for an incompressible liquid:

$$\rho(\partial \mathbf{u}/\partial t + (\mathbf{u}\cdot\nabla)\mathbf{u}) + \nabla p - \mu\nabla^2\mathbf{u} = 0,$$
$$\nabla\cdot\mathbf{u} = 0, \quad (1)$$

where ρ, μ, p, and $\mathbf{u}$ are density, viscosity, pressure, and velocity of the liquid, respectively. The effect of compressibility is neglected because the characteristic time of bubble growth and drop ejection ($\sim 10\ \mu$s) is long compared to the time of sound propagation to the inlet and the outlet of the nozzle.

Several boundary conditions are imposed in the calculation. At the inlet of the nozzle, the constant pressure boundary condition is imposed:

$$p = p_{\mathrm{res}}, \quad (2)$$

where p_{res} is the pressure of the ink reservoir. At the nozzle wall, the no slip boundary condition is imposed:

$$\mathbf{u} = 0. \quad (3)$$

At the outlet of the nozzle and the surface of the drop, the free surface boundary condition is imposed:

$$p = p_{\mathrm{amb}} + \sigma(1/r_1 + 1/r_2), \quad (4)$$

where p_{amb} is the atmospheric pressure, σ is the surface tension of the liquid, and r_1 and r_2 are principal radii of the surface. The contribution of viscosity at the surface is neglected (Harlow and Welch, 1965) because it does not have significant effect as long as the viscosity is not large. The shape of the surface is tracked by the volume of fluid (VOF) function defined at each computational mesh (Hirt and Nichols, 1981). At the bubble surface, a similar boundary condition is imposed:

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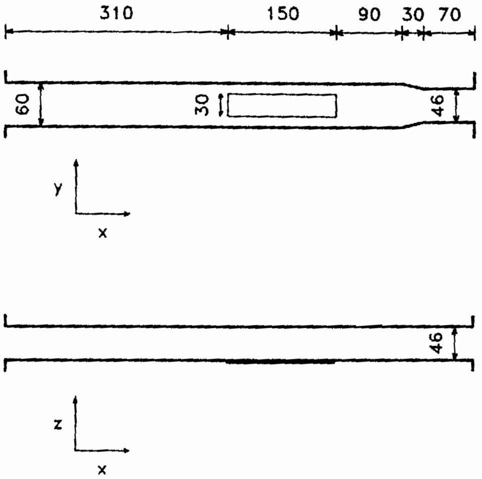

Fig. 1 Nozzle shape of a prototype bubble jet printer. Unit: μm.

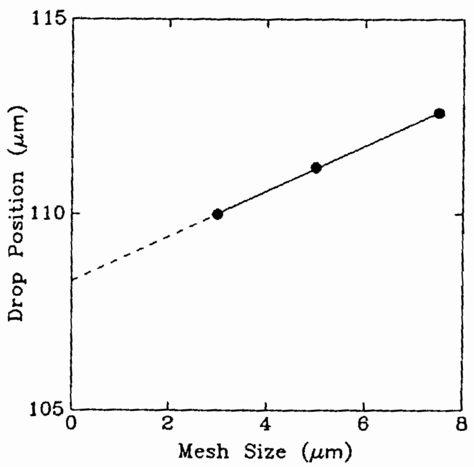

Fig. 2 Calculated drop position at t = 40μs, for various mesh size

$$p = p_v + \sigma(1/r_1 + 1/r_2), \tag{5}$$

where p_v is the bubble pressure described below.

Bubble Model. Since the ink in the nozzle is heated by an extremely high heat flux pulse ($\sim 10^8$ W m^{-2}) with a very short duration (1 ~ 10 μs), the bubble generation temperature is close to the superheat limit (~300°C, for water) and the initial bubble pressure is estimated to be very high (Asai et al., 1987; Asai et al., 1988; Asai, 1989; and Asai, 1991). The temperature and the pressure rapidly decrease in a few microseconds because the thickness of the heated ink is very thin. Thus the bubble pressure can be modeled by an impulsive function:

$$p_v \approx P\delta(t) + p_s, \tag{6}$$

where P is the pressure impulse and p_s is the bubble pressure in the later stage. Numerically, such impulsive pressure change can be described by a rapidly decreasing function of time t:

$$p_v = p_g f(t/t_0), \tag{7}$$

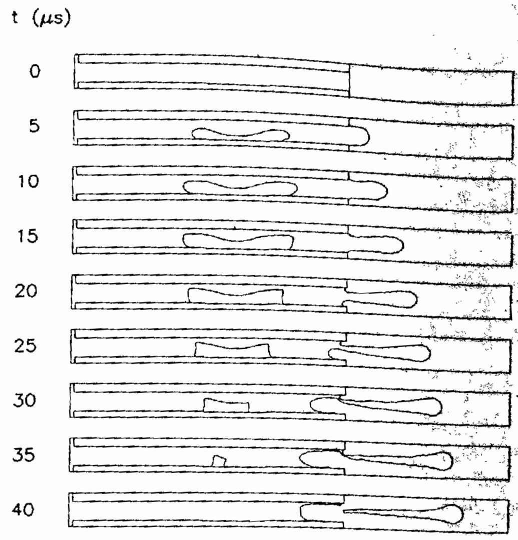

Fig. 3 Calculated cross section of bubble and drop in the xz plane, using 3μm mesh. p_g = 7.5MPa, t_0 = 0.17μs, μ = 4.5mPa s, σ = 50mN m^{-1}.

or bubble volume V:

$$p_v = \begin{cases} p_g g(V/V_0), & \text{for } dV/dt \geq 0 \\ p_s, & \text{for } dV/dt < 0 \end{cases} \tag{8}$$

where p_g is the initial bubble pressure, and t_0 and V_0 are constant parameters. In this paper the following function is used (Asai, 1991):

$$p_v = (p_g - p_s)\exp[-(t/t_0)^{1/2}] + p_s$$
$$p_g \gg p_s. \tag{9}$$

This expression has the property that $p_v = p_g$ at $t = 0$ and $p_v \approx p_s$ at $t \gg t_0$. The pressure impulse, P, and the work done by the bubble, W, are calculated by Eq. (9) as:

$$P \approx 2p_g t_0,$$
$$W \approx P^2/(2A_l), \tag{10}$$

where A_l is the inertance of the nozzle (Asai et al., 1987 and Asai, 1991).

Vectorization. The three-dimensional finite difference scheme requires a great amount of computation. We used vectorizable algorithms for CPU consuming loop calculations in order to reduce the CPU time. The most CPU consuming part of the calculation was the determination of the pressure field by the SOR method. Since the conventional SOR algorithm cannot be vectorized, we used the even-odd SOR technique.

The numerical calculation was done by the HITAC S820/60 super computer. The rate of vectorization was about 90 percent and the CPU time was reduced to about 1/7. The typical CPU time for a calculation was several minutes ~ several hours, depending on the number of mesh cells.

Nomenclature

A_l = inertance of the nozzle, kg m^{-4}
P = pressure impulse, Pa s
p = pressure, Pa
p_{amb} = ambient pressure, Pa
p_g = initial bubble pressure, Pa
p_{res} = reservoir pressure, Pa
p_s = bubble pressure in later stage, Pa
p_v = bubble pressure, Pa
r_1, r_2 = principal radii of the surface, m
S_h = area of heater, m^2
t = time, s
t_0 = time constant, s
$\mathbf{u}$ = velocity, m s^{-1}
V = bubble volume, m^3
V_0 = volume constant, m^3
W = work done by the bubble, J
μ = viscosity, Pa s
ρ = density, kg m^{-3}
σ = surface tension, N m^{-1}

Table 1 Composition of the ink used in the experiment

Ink	Water ratio [Mass %]	Density [g cm^{-3}]	Viscosity [mPa s]	Surface tension [mN m^{-1}]
A	55	1.0	4.5	53.8
B	29	1.0	11.8	49.0
C	9	1.0	16.7	45.7

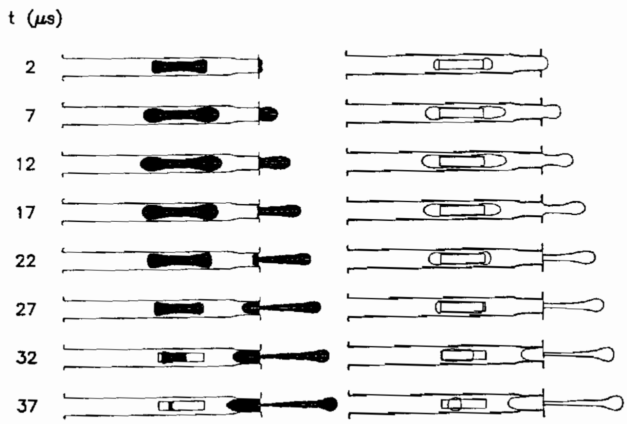

Fig. 4(*a*) Ink A: p_g = 7.5MPa

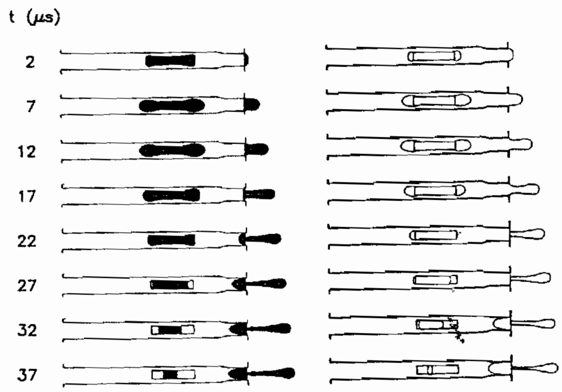

Fig. 4(*b*) Ink B: p_g = 7.3MPa

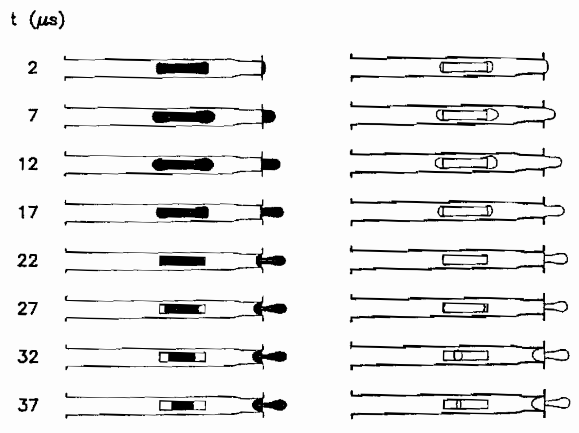

Fig. 4(*c*) Ink C: p_g = 6.6MPa

Fig. 4 Comparison of the calculated (left) and the experimental (right) top views of bubbles and drops

Results and Discussion

Mesh Size. In order to determine the appropriate mesh size for the calculation, the results for different mesh sizes are compared. The calculation was done for a nozzle shown in Fig. 1, which is a prototype nozzle for the BJ-80 bubble jet printer. The calculation was done only in the region $y \geq 0$ because of the symmetry of the nozzle. The initial bubble pressure, p_g, was taken to be 7.5MPa (saturated vapor pressure of water at ~300°C) and the time constant, t_0, which is estimated by the bubble dynamics (Asai et al., 1987 and Asai, 1991), was taken to be 0.17μs. The pressure p_s was chosen to be 16kPa and 101kPa (the saturated water vapor pressure for the ambient temperature and the boiling point), before and after the ink touch the heater surface, respectively. The boundary pressures, p_{res} and p_{amb}, are taken to be 101kPa.

The dependence of the calculated results on the mesh size is shown in Fig. 2. The discretization error can be estimated by extrapolating the result to the 0 μm mesh (dashed line in Fig. 2). In the following calculations, we use the 3 μm mesh, because the discretization error is estimated to be small enough (~2 percent) for this mesh size. The result of calculation for the 3μm mesh is shown in Fig. 3.

Comparison With an Experiment. In order to verify the validity of the calculation, the results are compared with an experiment (Asai et al., 1987) using three types of ink shown in Table 1 and the nozzle shown in Fig. 1. The time constant, t_0, was chosen to be 0.17μs, as above. The initial bubble pressure, p_g, was chosen to be 7.5, 7.3, and 6.6 MPa for ink A, B, and C, respectively, so that the calculated drop velocity agrees with the experiment. These values are reasonable considering the fraction of water in ink. The comparison of the calculation with the experiment is shown in Fig. 4. The drop is ejected for ink A and B, and is not ejected for ink C. The calculated shapes of the bubble and the drop agree well with the experiment.

Effect of Ink Property. The effect of the initial vapor pressure, p_g, viscosity, μ, and the surface tension, σ, on the volume and the velocity of the ejected drop is numerically investigated. The results are shown in Fig. 5. It is shown that the drop volume and velocity increase as the initial pressure increases and decrease as the viscosity or the surface tension increases.

The calculation shown above gives a basic guide to the design of ink, in that it predicts the effect of ink property on the volume and the velocity of the ejected drop, which greatly affect the printing quality (Shioya et al., 1989). Such calculation is very useful compared to the experiment, because each ink property (initial vapor pressure, viscosity and surface tension) cannot be changed independently in the experiment.

Application to the Nozzle Design. The three-dimensional calculation can be applied also to the design of the nozzle shape. In this case, the time constant, t_0, should be determined by an experiment or a theory (Asai, 1991). But once the value of t_0 or V_0 is determined for a nozzle, the calculation can be done for variety of nozzles, by changing the value so that the work done by the bubble to the ink, W, becomes approximately proportional to the size of the heater, S_h. If we use the bubble model (7), the value of t_0 should be adjusted proportionally to $(A_l S_h)^{1/2}$, because the work, $W \approx (\int p_v dt)^2/(2A_l)$, calculated by the model, is approximately proportional to t_0^2/A_l. If we use the bubble model (8), adjustment is simply done by changing the value of V_0 proportionally to S_h, because the work, $W = \int p_v dV$, calculated by the model, is approximately proportional to V_0.

The merit of the numerical study is that the nozzle shape is easily changed in the calculation, while the experimental study requires a lot of time and cost to make prototypes of many kinds. It is noted that the calculation will not produce enough information to the design of the nozzle shape, if it is done only outside the nozzle.

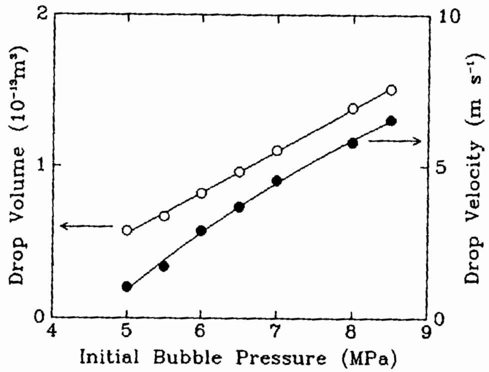

Fig. 5(a) Effect of initial bubble pressure

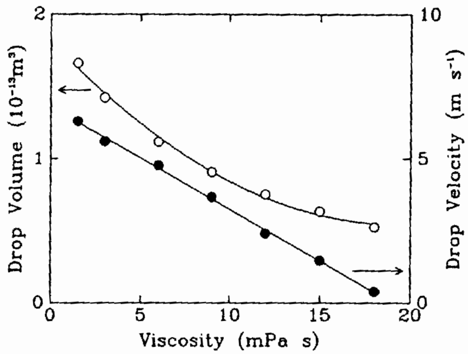

Fig. 5(b) Effect of viscosity

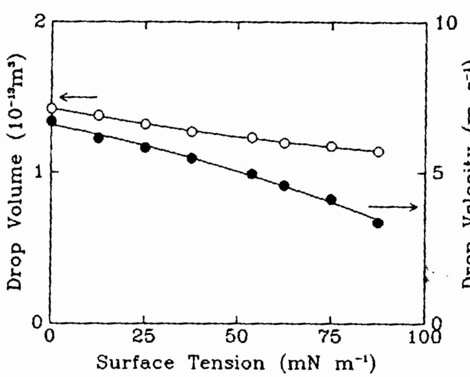

Fig. 5(c) Effect of surface tension

Fig. 5 Effect of ink property on the drop volume and the drop velocity

Conclusions

The numerical method to solve the three-dimensional Navier-Stokes equation for the motion of ink both inside and outside the nozzle of a bubble jet printer, presented in this paper, is successfully applied to the prediction of the bubble growth and the drop ejection phenomena. The results of calculation for three types of ink agreed well with experimental data. Further, the effect of ink property (initial bubble pressure, viscosity and surface tension) on the drop size and velocity is numerically investigated. It is concluded that the three-dimensional calculation is very useful to the design of bubble jet printers, because a lot of time and cost to make and evaluate prototypes are saved.

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