THIXOTROPIC FLOW EFFECTS UNDER CONDITIONS OF STRONG SHEAR

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Abstract

Semi-solid metal can be formed into useful products of near net shape and high quality. Use of this method has been limited, however, because of the difficulty of working with metal in the semi-solid state, which has strong shear rate, temperature and time-dependent properties. In this paper we describe a thixotropic model and apply it to the problem of small metal droplets impacting on a flat plate.

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Introduction

Semi-solid metal forming processes are currently the subject of considerable research because of the possibilities they offer to improve quality and reduce production costs for metal parts [1]. Using a semi-solid process, however, is challenging because metal in a semi-solid state exhibits complex rheological behavior that is not well understood.

Most often, a semi-solid material is characterized by its viscosity, which is observed to be a strong function of solid content and shear rate. To further complicate matters, the viscosity is generally observed to change with the period of time the material is subjected to, or not subjected to, shearing. Perhaps the best known examples of this phenomenon are ketchup and paint. Ketchup can be thinned by shaking (i.e., shearing) but thickens when allowed to sit for a time.

Paint must be thin enough to spread easily, thick enough not to run off a painted surface, then again thin enough for imperfections to be smoothed out by surface tension induced flow. These requirements can be met by a material that thins when sheared then thickens through a subsequent relaxation period. The relaxation period must be short enough to keep the paint from running, yet long enough to allow for smoothing to occur.

It is customary to use the expression "thixotropic" to describe this type of time-dependent behavior. An excellent and highly readable account of common thixotropic materials is given in Ref. [2].

In this paper we describe a generic thixotropic flow model that has been incorporated into a commercial computer program, FLOW-3D, for the simulation of fluid flow and heat transfer [3]. To illustrate how this type of program can be used to explore the behavior of thixotropic materials, it has been applied to the problem of metal drop impact on a flat surface.

Through a series of computational experiments we investigate how different viscosities and thixotropic relaxation rates alter the impact behavior of droplets. One goal of these experiments is to see if droplet impact might provide a simple, qualitative test of thixotropic properties of a material. However, the study of metal droplet impacts also has intrinsic interest for it is an important feature in processes such as plasma coating and micro soldering.

The Thixotropic Model

To be useful, a thixotropic flow model must have some basic features that are observed to exist in real materials. The model proposed here has been constructed to have three essential ingredients.

First, the existence of a relaxation rate for fluid viscosity implies the necessity of tracking the history of individual fluid particles. This can be done in an Eulerian flow representation by constructing a transport equation for the viscosity.
Second, observations suggest that the rate at which fluids are thinned by shear and the rate they thicken in the absence of a shear can be quite different. Thus, we want to add to the transport equation terms describing thinning and thickening processes having differing rates.

Finally, it is generally conceded that a viscosity versus shear-rate relation can be constructed for thixotropic materials under conditions of constant shear. That is, if the material is subjected to a constant shear rate for a long enough time, it will come to an equilibrium value of viscosity. In most cases these viscosity versus shear-rate relations have a power-law-like dependence. It is assumed here that the equilibrium relationship is a known function, \( \mu_e \) of temperature (or solid fraction) and shear rate.

We propose the following transport equation having the above three properties:

\[
\frac{\partial \mu}{\partial t} + \vec{u} \cdot \nabla \mu = \alpha \text{Min}(\mu_e - \mu, 0) + \beta \text{Max}(\mu_e - \mu, 0)
\]  

(1)

where \( \mu \) is the viscosity (a function of space and time) and \( \vec{u} \) is the fluid velocity. The rate coefficients, \( \alpha \) and \( \beta \), are the relaxation rates for thinning and thickening respectively. By definition "thinning" means \( \mu \) is greater than \( \mu_e \) and is trying to relax to the lower equilibrium value. Conversely, "thickening" means \( \mu \) is relaxing toward a larger equilibrium value.

A typical situation would be for a fluid particle to be thinning because of an applied shear; but when the shear stress is suddenly reduced, the equilibrium viscosity changes to a larger value and the fluid particle then begins to thicken.

In the present case, we assume that the relaxation coefficients \( \alpha \) and \( \beta \) are constants, but there is good evidence that the thinning coefficient should depend on shear rate (e.g., a linear dependence makes some physical sense).

Use of a transport equation for \( \mu \) differs from most other approaches, for example, those based on the Moore model in which the internal structure of a thixotropic fluid is represented by a single variable \( \lambda \). A transport equation for \( \lambda \) is constructed that includes terms describing the buildup and breakdown of the internal structure [4].

Our transport equation for \( \mu \) can be directly related to the Moore model [5]. However, a transport equation for \( \mu \) is more convenient for inclusion in a fluid dynamics simulation program, which requires a value for \( \mu \). Also, the \( \mu \) transport equation directly incorporates the measured equilibrium viscosity into the model. These two facts, coupled with the familiar concept of relaxation rates, makes the present approach easy to use and understand.

In the next section we apply this model to the problem of liquid metal droplets hitting a flat plate. This problem possesses large, transient shear rates, and it is an easy situation to test experimentally. Comparison of calculations using different values of the relaxation times clearly shows how changing thixotropic properties can dramatically alter the resultant droplet flow.
A Droplet Impact Test Problem

Let us suppose a 4 mm diameter drop of Tin containing 15% Lead (by weight) is allowed to fall from rest onto a plate located 6 cm below the drop. The droplet speed at the time of impact is 108.44 cm/s.

Further, suppose the drop is heated to a temperature of 197 °C that is between the liquidus and solidus temperatures (i.e., between 183 and 211 °C) so that it has approximately a 47% solid content.

The plate is given a constant temperature of 20.0 °C and a heat transfer coefficient with the liquid metal of 1.0E+7 gm/s²/C. Using this value we estimate the time interval needed to cool 10% of the droplet material to the solidus temperature to be roughly 37 ms, which is about ten times longer than the time to complete the impact (i.e., longer than \( t = \text{diameter/impact-speed} \)). For this reason, and for simplicity, we shall neglect temperature-dependent effects in the equilibrium viscosity model.

As the metal temperature nears the solidus limit, metal is allowed to solidify, but this is a slow process that does not influence the basic flow dynamics of the early-time impact.

We selected the Tin-Lead material for this study because there is published data for its thixotropic behavior [6]. The equilibrium viscosity is fit with a Carreau model having the form,

\[
\mu_E = \mu_\infty + \frac{\mu_0 - \mu_\infty}{1 + \left(\frac{\dot{\varepsilon}}{1.12}\right)^{0.465}}
\]

where \( \dot{\varepsilon} \) is the strain rate, \( \mu_0 \) the viscosity at zero strain rate and \( \mu_\infty \) the asymptotic viscosity as the strain rate approaches infinity.

Here we have used a zero shear-rate viscosity of 200 gm/cm/s and an asymptotic viscosity at infinite shear rate of 0.65 gm/cm/s. Note that at high shear rates the viscosity decreases inversely with the shear rate to the power 0.93.

The Numerical Model

All simulations were performed with the commercial software package FLOW-3D developed by Flow Science, Inc. of Los Alamos, NM. This program solves the time-dependent and three-dimensional Navier-Stokes equations coupled with heat transfer. A host of auxiliary models are also available for including the effects of surface tension, solidification, viscous heating and other processes likely to be important during droplet impact [3]. The thixotropic model was added to the program for this work and will be available in future releases of the code.

Axisymmetry is assumed. The flow region extends 1.0 cm radially and 0.5 cm vertically. This region is subdivided by a fixed grid of rectangular control volumes with 40 cells in the radial direction and 20 cells in the axial direction.
At the beginning of the computation a spherical liquid drop was placed with its lower surface slightly above the plate and moving downwards with the free-fall velocity of 108.44 cm/s.

Representation of the liquid in the fixed grid is accomplished with the Volume-of-Fluid method [7]. Zero tangential stress conditions are applied at the free surface of the drop. The normal stress condition includes a surface tension pressure corresponding to a surface tension value of 100.0 gm/s² and the contact angle between the metal and the plate was 90°.

**Results of Simulations**

In order to show how thixotropic effects alter the impact behavior of a metal droplet, we shall compare four computations. The first example is a limiting case in which the material viscosity is constant at the initial viscosity of the Tin-Lead drop. This corresponds to a zero thinning relaxation rate.

In a second limiting case the viscosity was allowed to immediately assume the shear-rate dependent, equilibrium viscosity value. This case corresponds to infinite relaxation rates for thinning and thickening.

The impact dynamics are completely different in these two cases. Consequently, the final two cases, having finite thinning and thickening rates, are most interesting because they show how thixotropic effects produce an intermediate flow behavior that might be used as a qualitative measure of relaxation rates.

**Zero Relaxation Rates**

Evaluation of the ratio of estimated forces associated with droplet impact gives us some idea of what to expect in the current tests. The ratio of inertia to viscous forces is the Reynolds number, Re. In this case, the Reynolds number at the instant of impact, Re=0.86, indicates that viscous stresses are strong, and there will be little distortion of the droplet during impact.

The ratio of gravitational forces to those of surface tension after impact is measured by the Bond number, Bo. This number, which is independent of viscosity, is Bo=6.2 for the present case. This value is low enough to indicate that surface tension is important and will have significant influence on the final drop shape, provided enough time is allowed for the surface tension forces to act.

Calculational results for the constant viscosity case are shown in Fig.1 (and in the first column of Fig.2), and are generally seen to confirm our expectations. The drop strikes and sticks on the plate. Wall-shear stress restricts flow on the surface of the plate causing the upper portion of the drop to mushroom out over a central stem. Velocities in the drop after impact mostly reflect surface tension adjustments until the final time shown, 0.1 s. At this time the drop has a more uniform curvature, and gravitational effects begin to dominate the flow, causing the outer edges of the drop to sag onto the plate.
This type of viscous drop impact can be easily reproduced in the kitchen using natural honey.

**Infinite Relaxation Rates**

Typical shear rates expected in a droplet impact can be estimated from the impact velocity divided by some fraction of the droplet diameter. In this case the impact speed is 108.44 cm/s and if we assume only about 20% of the drop radius participates in the initial distortion of the drop, the local rate of strain will be on the order of 2700/s, a rather large value. According to Eq.2, at this value the equilibrium viscosity is reduced from its initial value of 200 to about 1.0 gm/cm/s, and so we expect considerably more flow to occur in this case.

The computational results obtained when the flow is governed by the non-Newtonian viscosity, Eq.2, is shown in column two of Fig.2. Shear thinning during the initial impact allows the droplet to jet out radially; but as the shear stress is reduced with time, the viscosity of the drop increases and eventually brings the drop to rest.

The drop cannot easily retract under the action of surface tension because the larger radius of the edge of the drop makes this force smaller and the viscosity has nearly returned to its initial large value. Chances are that it will solidify before surface tension can pull it together.

This type of flat, circular splat is typical of ketchup and paint.

**Finite Relaxation Rates**

Relaxation rates for the Sn-Pb material are unknown, but for present purposes we know that the impact time scale is on the order of 1 to 2 ms, so that we must use characteristic times of this order if we are to see the effects of a finite relaxation time. Therefore, for illustrative purposes we have selected a thinning rate of 1000/s, corresponding to a 1.0 ms characteristic time.

In the third column of Fig.2 are the computed results using both a thinning and a thickening rate of 1000/s, while in the fourth column only the thinning rate is 1000/s and the thickening rate is nearly zero at 0.0001/s.

For the first 10 ms following impact, the two finite-rate calculations have nearly identical droplet shapes. This was expected because the droplets were thinning at the same rate in both cases. However, after 10 ms almost complete thickening to the initial state has occurred in the drop with the 1.0 ms characteristic thickening time, while in the other case almost no thickening has occurred.

Neither drop flows out radially as far as the previous case with infinite relaxation rates because the viscosity does not have time to thin out completely. By 50 ms the example with approximately zero thickening rate has a maximum viscosity of only about 32.2 gm/cm/s and, consequently, has been able to retract under the action of surface tension into a rounder shape.
These results show how the resultant droplet shape depends on its thixotropic behavior. Changing the initial viscosity, drop size and height, or a variety of other parameters such as contact angle, surface roughness and heat-transfer rate would likely produce different shapes. Computational simulations offer a convenient way to explore options that give the best results for a particular application. Since each computation presented here only required a couple of minutes of CPU time, many simulations can be performed at little cost.

Summary

We have described a simple transport equation model for the time-dependent viscosity of thixotropic materials. This model is convenient for inclusion in computational fluid dynamics programs and has the added advantage that it uses readily measured input data.

The proposed model has been applied to the problem of liquid metal drops impacting on a rigid plate. Heat transfer, viscous heating and solidification effects were included in the analysis to show the scope of effects that can be included in numerical simulations. However, heat transfer played a negligible role in the time periods simulated.

The principal objective of this study was to show how the degree of thixotropy influences impact flow behavior. Our results indicate that droplet shapes after impact are significantly influenced by their thixotropic behavior. This result suggests the possibility of using simple drop experiments to qualitatively estimate relaxation rates.

References


Fig. 1. Impact of viscous drop. Distance units are cm. Times in ms from top to bottom 0, 1, 5, 50 and 100. Surface tension is dominating the flow in the 5 to 50 ms period; gravity is dominant by 100 ms.
Fig. 2. Comparison of thixotropic relaxation times. Times from top to bottom in ms are 1, 2, 5, 10 and 50. First (left) column has zero rates (constant viscosity). Second column has thinning and thickening rates of 1000/s. Third column has 1000/s thinning rate and 0.0001 thickening rate. Last (right) column has infinitely fast rates.