Surface Tension Induced Instability of Viscous Liquid Jets

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ABSTRACT

Viscous liquid jets are inherently unstable, and drop formation can be induced by various means including surface tension modulation. Furthermore, surface tension is temperature dependent, and can be modulated by locally heating the surface of the jet as it leaves an orifice [1]. Such modulation can be implemented using CMOS/MEMS technology by integrating heating elements into the orifice manifold. In this paper, we present an analytical analysis of the capillary instability of a viscous jet due to a sinusoidal modulation of its surface tension. We solve the linearized axisymmetric Navier-Stokes equations for a Newtonian fluid subject to the slender jet approximation, with a spatially periodic variation of surface tension imposed along the length of the jet. We derive a closed-form expression for the time dependence of the jet radius. This expression enables rapid parametric analysis of jet instability (time to drop formation) as a function of jet radius, velocity, viscosity, density, and surface tension.

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Keywords: Viscous jet instability, surface tension modulation, capillary instability of a viscous jet, drop formation, slender jet analysis.

1 INTRODUCTION

Although considerable work has been done on the temporal instability of viscous liquid jets, few authors have studied jet instability caused by direct surface tension modulation [2]. In this paper, we present results for this phenomenon that are apparently new. Specifically, we study the dynamics of drop formation for a jet exiting an orifice with a time-harmonic heat pulse applied to a heating element that surrounds the orifice (Fig. 1). This induces a time-harmonic modulation of the surface tension σ of the jet, which has an equation of state of the form

$$\sigma(T) = \sigma_0 - \beta(T - T_0) \tag{1}$$

where σ_0 is the surface tension at temperature T_0. Therefore, the physical problem is one in which a time-harmonic modulation is imposed on the surface tension of the jet at the orifice.

Instead of solving the physical problem directly, we solve an equivalent problem in which an infinite jet (no orifice) is subjected to a spatially periodic sinusoidal modulation of its surface tension along its length (Fig. 2). The spatial wavelength $\lambda = \mathrm{v}_0 \tau$ where v_0 is the velocity of the jet, and τ is the period of the time-harmonic modulation. It is well known that the time to drop formation (break-up length) of a high velocity jet emanating from a stationary nozzle is predicted remarkable well from the linear analysis of an infinite jet.

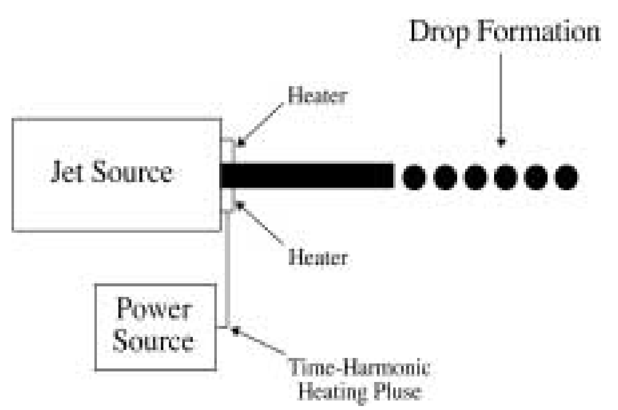

Figure 1: Physical system.

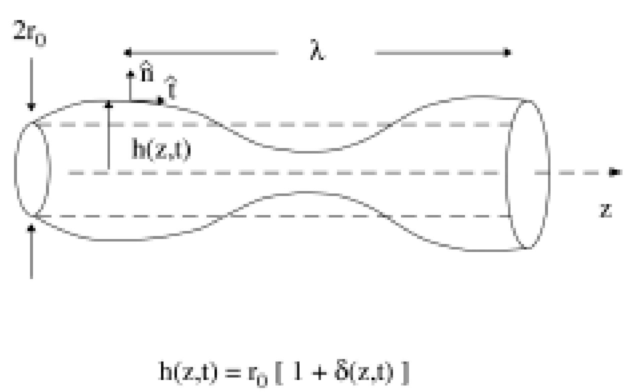

Figure 2: Infinite axisymmetric jet.

2 THEORY

We consider the dynamic behavior of an infinite, viscous axisymmetric jet of Newtonian fluid that is subjected to a spatially periodic sinusoidal modulation of its surface tension. The equations governing the behavior of a slender axisymmetric jet are as follows:

Navier-Stokes:

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (rv_r)}{\partial r}\right)+\frac{\partial^2 v_r}{\partial z^2}\right] \quad (2)$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial z}\right)+\frac{\partial^2 v_z}{\partial z^2}\right] \quad (3)$$

Continuity:

$$\frac{1}{r}\frac{\partial (rv_r)}{\partial r}+\frac{\partial (v_z)}{\partial r}=0 \quad (4)$$

Boundary Conditions:

Normal Stress:

$$(\mathbf{T}\bullet\hat{n})\bullet\hat{n}=-2H\sigma \quad (5)$$

Tangential Stress:

$$(\mathbf{T}\bullet\hat{n})\bullet\hat{t}=\hat{t}\bullet\nabla_s\sigma \quad (6)$$

Kinematic (at jet surface):

$$\frac{D}{Dt}\left(r_s-h(z,t)\right)=0 \quad (7)$$

On axis (r = 0):

$$v_r=0 \quad and \quad \frac{\partial v_z}{\partial r}=0 \quad (8)$$

where v_r and v_z are the radial and axial velocity of the fluid in the jet, respectively, $h(z,t)$ defines the radial position of the boundary surface of the jet, $\hat{n}$ and $\hat{t}$ are unit vectors normal and tangential to the surface of the jet, ∇_s is the gradient operator along the surface of the jet, σ (z,t) is the surface tension of the jet, and $\mathbf{T}$ is the stress tensor. The function H is given by

$$H=\frac{1}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) =\frac{1}{2}\left(\frac{1}{h(1+h'^2)^{1/2}}+\frac{h''}{(1+h'^2)^{3/2}}\right) \quad (9)$$

where R_1 and R_2 are the radii of curvature of the surface of the jet, and $h'=\frac{\partial h}{\partial z}$. We expand $v_z(r,z,t)$ and $p(r,z,t)$ in a powers of r [3],

$$v_z(r,z,t)=v_0(z,t)+v_2(z,t)r^2+\ldots \quad (10)$$

$$p(r,z,t)=p_0(z,t)+p_2(z,t)r^2+\ldots \quad (11)$$

From (10) and the continuity condition (4) we obtain an expansion for $v_r(r,z,t)$,

$$v_r(r,z,t)=-\frac{\partial v_0(z,t)}{\partial z}\frac{r}{2}-v_2(z,t)\frac{r^3}{4}+\ldots \quad (12)$$

Using these expansions in (2)-(7), and collecting the lowest order terms, we obtain

$$\frac{dv_0}{dt}+v_0\frac{\partial v_0}{\partial z}=-\frac{2}{\rho}\frac{\partial (H\sigma)}{\partial z} +\frac{2}{\rho h}\frac{\partial \sigma}{\partial z}+\frac{3\mu}{\rho h^2}\frac{\partial}{\partial z}\left(h^2\frac{\partial v_0}{\partial z}\right) \quad (13)$$

and

$$\frac{\partial h}{\partial t}=-v_0\frac{\partial h}{\partial z}-\frac{h}{2}\frac{\partial v_0}{\partial z} \quad (14)$$

These equations can be reduced further using the following expansions

$$v_0(z,t)=\mathrm{v}_0+v(z,t) \quad (15)$$

$$h(z,t)=r_0[1+\delta(z,t)] \quad (16)$$

$$\sigma(z,t)=\sigma_0+\sigma_1(z,t) \tag{17}$$

where r_0, v_0 and σ_0 are the unperturbed radius, velocity and surface tension of the jet, and $v(z,t)$, $\delta(z,t)$ and $\sigma_1(z,t)$ are perturbations to these values. We solve for $v(z,t)$ and $\delta(z,t)$, but we impose a specified perturbation $\sigma_1(z,t)$.

To simplify the analysis, we choose a coordinate system at rest with respect to the jet. Specifically, we use the Galilean transform

$$\eta = z-\mathrm{v}_0\, t \tag{18}$$

Using this coordinate, the modulation of the surface tension is of the form

$$\sigma_1(\eta)=-\frac{\Delta\sigma}{2}\left(1+\cos(\frac{2\pi}{\lambda}\eta)\right) \tag{19}$$

where $\Delta\sigma$ is the maximum change in surface tension. We apply (15), (16), (17), and (18) to (13) and (14), and then linearize the resulting equations by ignoring products of derivatives etc. as discussed by Lee [4]. This gives,

$$\frac{\partial v}{\partial t}-\frac{\sigma_0}{\rho r_0}\left[\frac{\partial\delta}{\partial\eta}+r_0^{\,2}\frac{\partial^3\delta}{\partial\eta^3}\right]-\frac{3\mu}{\rho}\frac{\partial^2 v}{\partial\eta^2}=\frac{1}{\rho r_0}\frac{\partial\sigma_1}{\partial\eta} \tag{20}$$

and

$$\frac{\partial^2 v}{\partial\eta^2}=-2\frac{\partial^2\delta}{\partial\eta\partial t} \tag{21}$$

Lastly, apply $\dfrac{\partial}{\partial\eta}$ to (20), and then the substitute (21) into the resulting equation and obtain

$$\frac{\partial^2\delta}{\partial t^2}+\frac{\sigma_0}{2\rho r_0}\left[\frac{\partial^2\delta}{\partial\eta^2}+r_0^{\,2}\frac{\partial^4\delta}{\partial\eta^4}\right]-\frac{3\mu}{\rho}\frac{\partial^3\delta}{\partial\eta^2\partial t}=-\frac{1}{2\rho r_0}\frac{\partial^2\sigma_1}{\partial\eta^2} \tag{22}$$

Equation (22) defines an initial-value problem for $\delta(\eta,t)$. We solve this subject to the initial conditions

$$\delta(\eta,t)=0 \qquad \textit{and} \qquad \frac{\partial}{\partial t}\delta(\eta,t)=0$$

The solution is

$$\delta(\eta,t)=\frac{\Delta\sigma}{2\sigma_0}\frac{1}{\left[1-r_0^{\,2}\left(\frac{2\pi}{\lambda}\right)^2\right]} \times\left\{1-\left[\alpha^-\frac{e^{\alpha^+ t}}{\left(\alpha^--\alpha^+\right)}+\alpha^+\frac{e^{\alpha^- t}}{\left(\alpha^+-\alpha^-\right)}\right]\right\}\cos\left(\frac{2\pi\eta}{\lambda}\right) \tag{23}$$

where

$$\alpha^\pm=\frac{-\frac{3\mu}{\rho}\left(\frac{2\pi}{\lambda}\right)^2\pm\sqrt{\left[\frac{3\mu}{\rho}\left(\frac{2\pi}{\lambda}\right)^2\right]^2+4\beta^2}}{2} \tag{24}$$

and $\beta^2=\dfrac{\sigma_0}{2\rho r_0}\left(\dfrac{2\pi}{\lambda}\right)^2\left[1-r_0^{\,2}\left(\dfrac{2\pi}{\lambda}\right)^2\right]$.

3 RESULTS

The analytical solution (23) enables rapid analysis of the time to drop formation. In the linear theory, drop formation occurs when $\delta(z,t)=1$. We demonstrate (23) by computing the time to drop formation as a function of viscosity and percent modulation $\Delta\sigma/\sigma_0$ for a jet with $r_0=5\mu m$, $\mathrm{v}_0=10\ m/s$ and $\lambda=40\,\mu m$ (Fig. 3). The analysis shows that the time to drop formation increases with viscosity and decreases with modulation as expected.

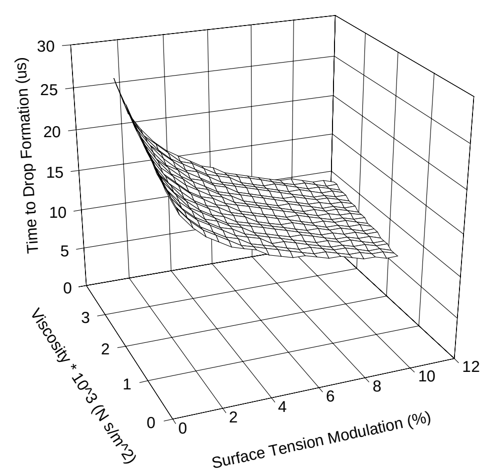

Figure 3: Parametric analysis of jet instability.

It is important to note that $\eta = 0, 2\pi, 4\pi, \ldots$ corresponds to points of maximum heating, or a maximum change in surface tension. An analysis of (23) shows that the jet necks at these points, and balloons at the points. $\eta = \pi, 3\pi, 5\pi, \ldots$. Therefore, we find that the jet necks at the heated regions and balloons at the cooler regions. We verified this result using CFD. Specifically, an axisymmetric analysis was performed using the FLOW-3D software [5]. A simulation sequence is shown in Figs. 4-6. The analysis confirms necking and ballooning of the jet at the heated and cooler portions of the jet, respectively.

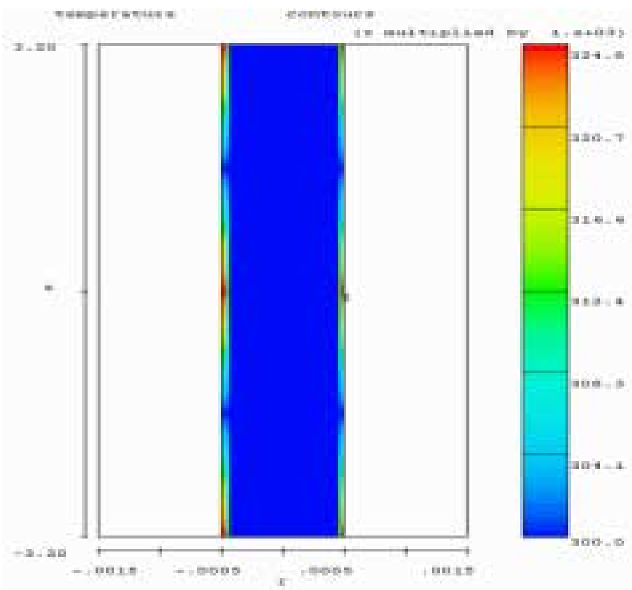

Figure 4: Initial state of jet with periodic heating applied.

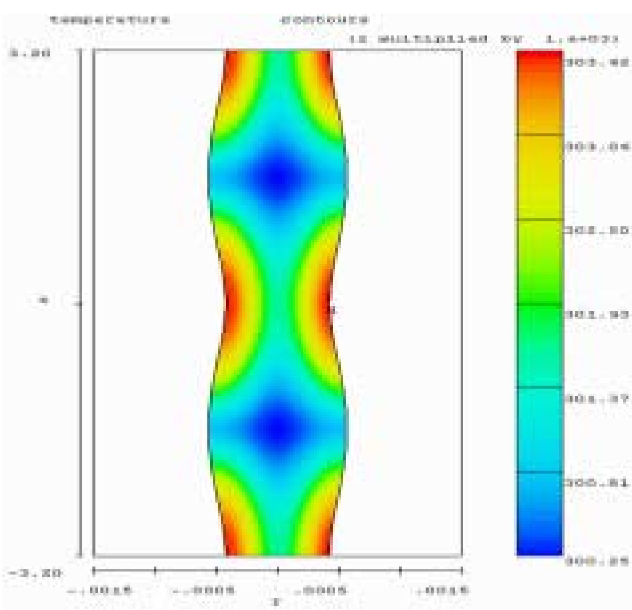

Figure 5: Onset of necking and ballooning.

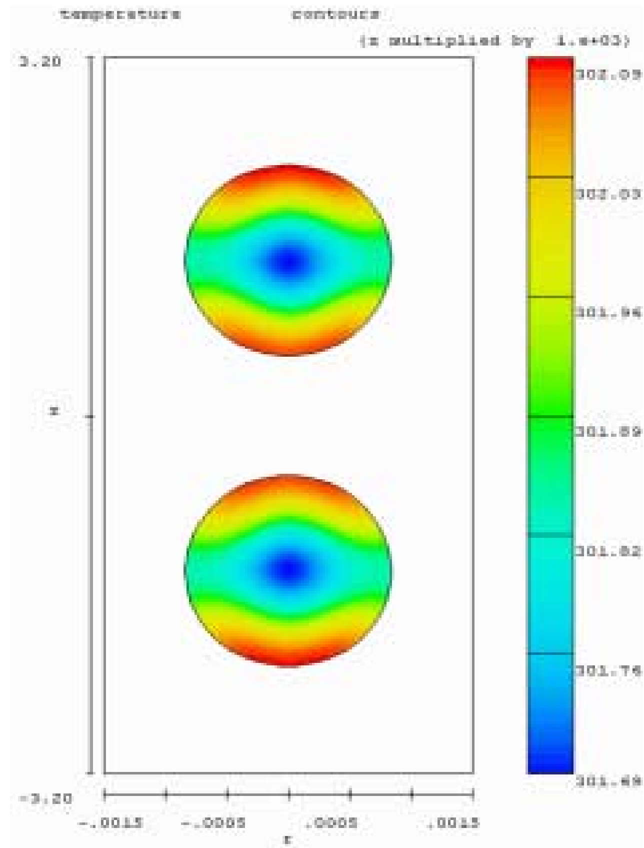

Figure 6: Final state with formed drops.

ACKNOWLEDGMENT

The authors would like to thank Professor Osman Basaran for numerous insightful technical discussions.

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[5] FLOW-3D is CFD software sold by Flow-Science Inc., Los Alamos, New Mexico 87544.