Simulation of Wave Impact Pressure on Vertical Structutes with the SPH Method

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***Abstract*— Such is the complexity of wave-structure interaction phenomena that most available theories and procedures for the design of maritime structures – both vertical and rubble mound breakwaters - are still derived by fitting simple conceptual models to the results of physical experiments, either in the field or in the laboratories. Recent developments of CFD techniques are now supplying a better understanding of the hydrodynamic phenomena involved. In particular, Eulerian Navier Stokes codes with special free surface treatment have reached a mature stage, as in [1], [2] so that operational design tools based on such techniques seem to be not too far away in the future, at least for low slope structures. Vertical and near vertical structures, however, seem to pose a bigger challenge since simulating wave impact on a steep wall, with its related sudden changes of the free surface, is a difficult task to achieve with classical Eulerian techniques. This is a sector where Lagrangian hydrodynamic methods may offer useful solutions. The Authors' past experience in SPH code efficiency and reliability analysis [3], [4], together with their long standing experience in maritime wave problems [5], [6], provide the background for the work to be presented at the workshop. Their own SPH parallel code is applied to investigate time and space distribution of wave pressure on a vertical marine structure. Results are then compared with the ones obtained from the Eulerian based program Flow 3D, showing a good agreement.**

I. INTRODUCTION

Coastal structures may be object of severe pressures induced by breaking waves. Typically, such dynamic actions consist of short, high impulses, reaching up to hundreds of kN/m^2, followed by longer, smaller hydrodynamic forces [7]. Prediction of such evolving pressures is not straight: estimates are given by several semi-empirical formulas, such as in [8], [9]. Their main disadvantage is due to their extremely sensibility to the particular situation they are referred to. In addition, wave impact pressures are influenced by the amount of entrapped air as shown in [8].

Recently, thanks to both the increasing power of computers and the reliability of numerical techniques, Computational Fluid Dynamics yields a better understanding of the hydrodynamic processes involved into breaking waves over seawall structures. Eulerian based schemes, discretizing the well known Shallow Water equations (SWE), give satisfactory results, at least for low seabed slops. On the other hand, impact on a vertical or structures of travelling waves over steep bottoms involves the propagation of not gradually varied flows. In such cases, depth averaged Navier Stokes (N-S) equations, that is SWEs, seem to be improper to accurately reproduce such phenomena.

Lagrangian description of N-S equations by means of the Smoothed Particle Hydrodynamics (SPH) technique [10], [11], overcomes the above mentioned difficulties. Its capability to well reproduce rapidly varied free surface flows has been proved over the past few decades [12]-[14]. A brief overview of the method is presented in the following. A comprehensive review of its theoretical aspects has recently been published [15], so they need not be fully discussed here.

II. THE SPH METHOD

With this method, the original physical domain is discretized by a finite number of particles which represent small volumes of the system. They move in response to forces such as gravity and pressure, carrying at the same time physical properties. A generic scalar or vector field f_i, at a given position $\underline{r}_i$, taken by the fluid particle "i", is given by a smooth interpolation from the values f_k carried by those N_n particles within the interaction sphere of radius r_c =2h [16]:

$$f_i = \sum_{k=1}^{N_n} f_k \cdot W\left(\left|\underline{r}_i - \underline{r}_k\right|, h\right) \qquad (1)$$

where "W" is the so called kernel or weighting function and "h" is known as the smoothing length. Here the cubic spline devised by Monaghan and Lattanzio [17] is adopted:

$$W(q) = A_n(n_d)\cdot\begin{cases} \frac{2}{3} - q^2 + \frac{1}{2}q^3 & 0 \le q < 1 \\ \frac{1}{6}(2-q)^3 & 1 \le q < 2 \\ 0 & q \ge 2 \end{cases} \tag{2}$$

being $q = |\underline{r}_i - \underline{r}_k|/2h = |\underline{r}_{ik}|/2h$ the dimensionless relative distance between the neighbouring particles "i" and "k". Same procedure is adopted for spatial operators, such as the divergence $\nabla\cdot$ or the gradient ∇.

The non-linear partial differential Equations of Navier-Stokes, are approximated by replacing the derivative operators with equivalent integral operators that are in turn discretized on the particle location [18], yielding the following set:

$$\frac{D\rho_i}{Dt} = \sum_{i=1}^{N} m_j\left(\underline{v}_i - \underline{v}_j\right)\cdot\nabla_i W_{ij} \tag{3.a}$$

$$\frac{D\underline{v}_i}{Dt} = -\sum_{j=1}^{N} m_j\left(\frac{p_j}{\rho_j^2} + \frac{p_i}{\rho_i^2} + \Pi_{ij}\right)\nabla_i W_{ij} + \underline{g} \tag{3.b}$$

where ρ, p, and $\underline{v}$ represent respectively density, pressure and velocity fields, $\underline{g}$ is the acceleration of gravity. Viscous shear is here modeled with the artificial viscosity Π proposed by Monaghan [15]. An equation of state is needed to close the model "(3)". The following stiff function [19] $p(\rho)$ has been implemented:

$$p_i = B\left[\left(\frac{\rho_i}{\rho_0}\right)^{\gamma} - 1\right] \tag{4}$$

where $\gamma = 7$ and "B" is function of the Mach number taken to be 0.1.

Time integration is carried out with a predictor-corrector scheme with a timestep "Dt" controlled by the CFL stability limit.

III. TEST CASE

The ability of SPH to numerically reproduce pressures arisen from waves' impact on a seawall structure is here investigated. A typical situation is sketched in Fig. 1, with a still water depth at the structure $d_s = 3.0$m, and a bottom slope $\tan\alpha = 1/20$. The resulting system is let evolve for an overall time t = 40 sec.

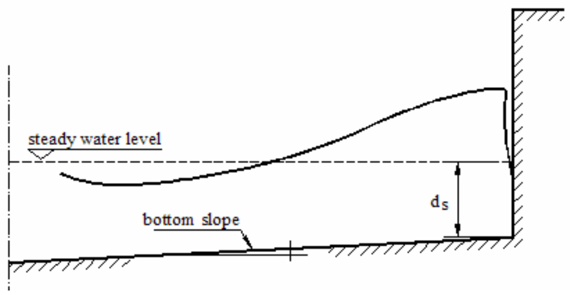


Figure 1. A breking wave over a vertical seawall structure.

About 70.000 computing particles are placed according to the initial steady water level (Fig. 2). Solid boundaries are modeled by a collection of fixed particles exerting on approaching moving ones a repulsive force [20] given by the following expression:

$$\underline{F}_{ik}(\underline{r}_{ik}) = \begin{cases} D\left[\left(\frac{2h}{|\underline{r}_{ik}|}\right)^{P_1} - \left(\frac{2h}{|\underline{r}_{ik}|}\right)^{P_2}\right]\frac{\underline{r}_{ik}}{|\underline{r}_{ik}|} & |\underline{r}_{ik}| < 2h \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where "D", "p_1" and "p_2" are fixed values according to according to [16]. This is to prevent particle penetration through the domain edges.

Waves are generated by means of a paddle located at a distance of L = 20m from the seawall. The implemented wavemaker moves following a sinusoidal form. In order to prevent numerical divergences, peaks are initially dumped by introducing an exponential function. The complete form is then given by:

$$x_p(t) = x_0 + A\cdot\left[1 - \exp(\lambda\cdot t)\right]\cdot\sin(\omega\cdot t) \tag{6}$$

where $x_0 = 1.6$m is the initial position of the paddle along the "*x*" axis, A = 0.5m is the amplitude, λ =-0.1 is the dumping coefficient and ω = 10rad/sec is the frequency.

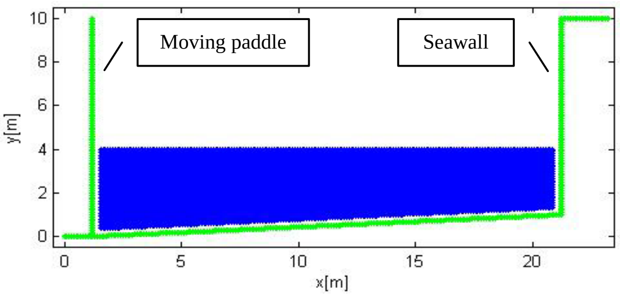


Figure 2. Physical domain – Initial condition.

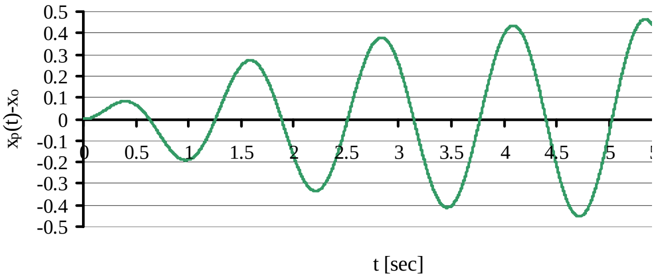


Figure 3. Paddle initially moves with lower amplitudes in order to prevent numerical divergences.

As shown in Fig. 3, amplitudes reach a steady value after few seconds. In order to correctly reproduce the same moving boundary condition on Flow 3D ® environment [21], which is an Eulerian based code, a column of boundary cells is placed at x_0 = 1.6m (Fig. 4) with a velocity law inferred from (6), as follows:

$$v(t)_{x0} = A\cdot\omega\cdot\cos(\omega\cdot t)\cdot[1-\exp(\lambda\cdot t)] - \lambda\cdot\exp(\lambda\cdot t)\cdot\sin(\omega\cdot t) \tag{7}$$

Peculiar aspects concerning pressure detecting on the seawall are next discussed.

IV. PRESSURE MEASUREMENT

Rather than considering “(4)” as a straight mean to deduct pressure distribution on the seawall, here authors perform a smooth interpolation in the local neighbourhood, according to “(1)”:

$$p_i = \sum_{k=1}^{N_n} p_k \cdot \alpha_{cut} \cdot W(|r_i - r_k|, h) \tag{8}$$

The introduced constant inside “(8)” is due to the compact support cut by half (Fig.5) for straight boundaries.

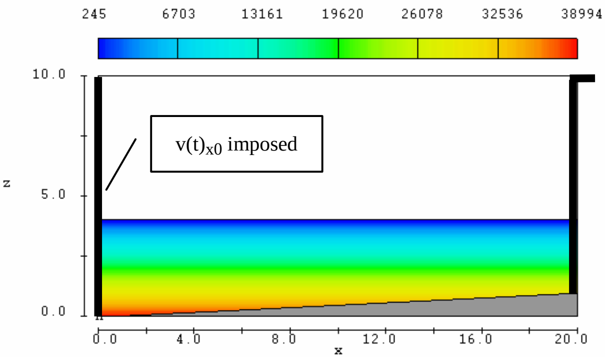


Figure 4. Initial conditions in Flow 3D ® environment. Legend refers to the pressure contour.

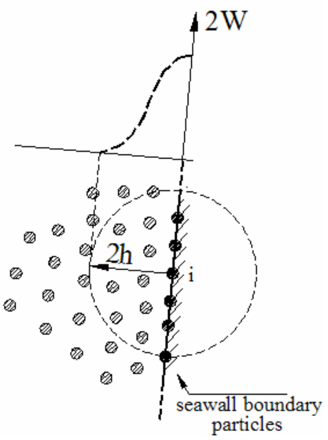
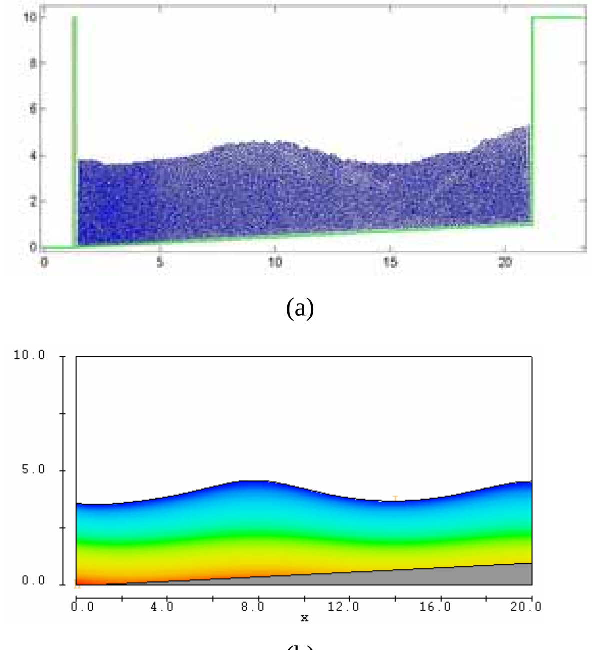


Figure 5. Smoothing interpolation of pressure field over half the compact support.

No contribution indeed comes from outside since there are no particles beyond the seawall. The advantage of “(8)” is that the spatial interpolation yields better results. Pressure field is, in other words, smoothed due to the finite system resolution. Results are shown in the following paragraph.

V. TEST RESULTS

The authors have carried out simulations of the forementioned case with their own developed parallel SPH-based code. At the same time, the same problem has been investigated with the software Flow 3D ®. In the following water level surface has been compared for a certain time.



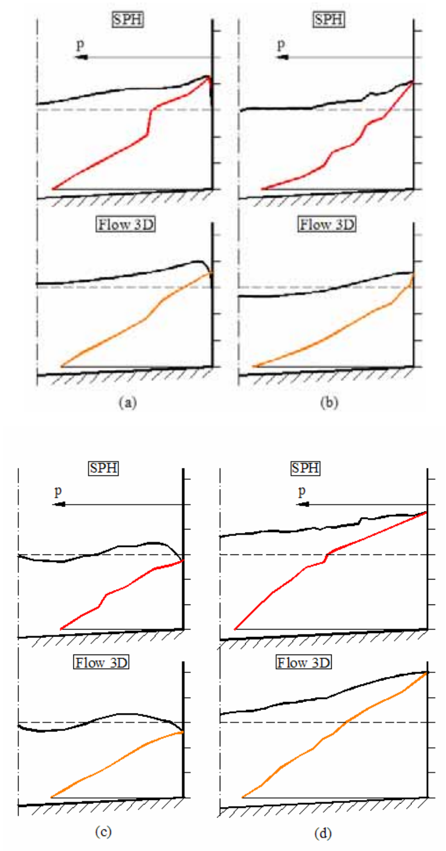
(a)

(b)

Figure 6. Evolving system al time = 20 sec. (a) SPH; (b) Flow 3D ®

Pressure distribution on the seawall due to waves' impact is in the following reported and compared with numerical results obtained with Flow 3D ®.

Results show how the pressure trend roughly follows the hydrostatic distribution; greater values have been detected near the free surface. This is due to the impact mechanism in which fluid is not gradually varied at the wall interface.



(a) (b)

(c) (d)

Figure 7. Pressure comparison along the seawall at times: (a) 10 sec, (b) 20 sec, (c) 30 sec, (d) 40 sec.

CONCLUSIONS

The Author's SPH parallel code is applied to investigate wave pressure distribution of on a vertical seawall structure. Results are then compared with the ones obtained from the Eulerian based program Flow 3D, showing a good agreement in terms of pressure distribution and free water level.

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