# Physical and Numerical Comparison of Flow over Ogee Spillway in the Presence of Tailwater

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**Abstract:** Data obtained from two physical models were compared to the results obtained from numerical model investigations of two ogee-crested spillways. In 2001, Savage and Johnson investigated ogee-crested spillways without the effect of tailwater; the present study includes the influence of tailwater on the spillway. The comparison showed that numerical modeling can accurately predict the rate of flow over the spillway and the pressure distribution on the spillway. The results of this study provide users of numerical models performance information that can be used to aid them in determining which tool to use to effectively analyze dams and their associated spillways.

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## Introduction

In the stability analysis of a dam, it is important to correctly identify and quantify all forces acting on a dam. Many of these forces can be computed using relatively simple methods such as calculating the hydrostatic upstream pressure or the gravity force due to the weight of the dam. However, when the force needs to be calculated in a region with accelerating or decelerating flow, calculation of the fluid pressure can be complex. Because of the complexity, for free flowing (unsubmerged) flow, the force acting on the dam crest and spillway is often considered nonsignificant because it is small compared to the other forces. But, when the downstream depth is increased and the spillway flow becomes submerged, then pressures on the spillway become significant and provide a resisting force to overturning or sliding. Noninclusion of this force results in a conservative estimate of dam stability. From a safety perspective, a conservative analysis is ideal but from an economic perspective it can be costly. On the other hand, assigning a full hydrostatic pressure to a submerged spillway may overestimate the pressure force thereby providing a false sense of security on the stability of the dam. Therefore, if a spillway encounters submergence, it is important that the spillway pressures be correctly determined. The purpose of this paper is to compare dam pressures calculated via a numerical model versus the pressures determined from a physical model. Comparing the two methods is a rigorous test that enables engineers to have more confidence in the numerical approach should they select it to analyze dams and their associated spillways.

In a previous study, Savage and Johnson (2001) compared the

discharge characteristics and pressure distribution for flows passing over an unsubmerged ogee spillway using a physical model, a numerical model, and information published in various design guides (Maynord 1985; USACE 1990; USBR 1977/1987). This study is an extension to the previous study with the focus being on the comparison of the pressure distribution on the spillway when tailwater is present. Because a physical model is considered the best available analysis tool, it was used as the baseline from which the other methods were measured against. However, because the flow changes from subcritical to supercritical and then back to subcritical as the discharge passes through a hydraulic jump or a drowned hydraulic jump, the hydraulics are more complex.

This paper provides information on how accurately a commercially available computational fluid dynamic (CFD) model can predict the spillway pressures when submergence occurs. It should be noted that there is a distinct difference between submergence of the spillway jet and dam submergence. Dam submergence occurs when the depth of the tailwater increases sufficiently to affect the discharge over the dam (USBR 1977/1987). This note deals primarily with jet submergence. The intent is to give engineers and potential users of numerical tools, more information about the performance and application of CFD model studies that may be used to assess the hydraulic performance and assist in the stability analysis of dams.

# Background

The pressures and discharge over an ogee-crested dam and spillway are dependent on the crest and spillway geometry, the upstream flow depth, and in the case of submergence—the downstream depth. In the current literature, there is significant information detailing the change in the discharge coefficient due to dam submergence. Harleman et al. (1963) compiled a bibliography on dam design that includes references to the change in the discharge coefficients due to dam submergence. However, there appears to be very little information outlining changes in spillway pressures due to submergence on the spillway prior to dam submergence.

Bradley (1945) defines four distinct types of flow over an

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ogee-crested dam. Type I flow is characterized by a supercritical jet of water staying attached through the length of the spillway. Type II flow is defined as true hydraulic jump forming on the spillway. Type III flow is defined as a drowned jump. A drowned jump is when the jet of water stays attached to the spillway face and does so for a considerable distance under the tailwater. This occurs when the tailwater depth is too great to allow a good hydraulic jump to form. Type IV flow occurs when the jet breaks up and the dam acts as a broad crested weir. The dam in this case, is generally under a high degree of submergence and the downstream depth becomes a significant variable in controlling the discharge. Shany (1950) reported that as the tailwater depth increases past total submergence for a Type IV flow, the pressure distribution on the downstream face of the dam approaches hydrostatic conditions. The results in this note primarily deal with Flow Types II and III.

For Flow Types II and III, there is no consensus on the extent that pressure force can be used in determining the stability of a dam. The U.S. Bureau of Reclamation (1977/1987) states that only the "minimum tailwater level associated with each reservoir level" be used to calculate the pressure force provided by the tailwater (Design Criteria 23; Design of Small Dams 318). Stelle et al. (1983) reported that they assumed the "full value of tailwater pressure" in calculating the stability of the concrete dam, meaning that the full hydrostatic pressure force as calculated using the depth of the tailwater was used. The U.S. Army Corps of Engineers (1995) is more conservative, indicating that as little as 60% of the tailwater depth should be used to calculate pressures on a dam when a significant hydraulic jump occurs. The U.S. Army Corps of Engineers (1995) also states that when the hydraulic jumps is reduced or eliminated that 100% of the tailwater depth can be used. This leaves the question of how much pressure force is on a submerged spillway and at what point is a drowned hydraulic jump sufficiently reduced or eliminated?

## **Physical Models**

For engineers to accurately answer the question pertaining to hydrodynamic force distribution on a dam and/or spillway, they must conduct a physical hydraulic model study, use guidelines provided by the U.S. Bureau of Reclamation or the U.S. Army Corps of Engineers, or perform a numerical study and solve the equations governing fluid flow over such structures. Although site specific physical models are considered as the best analysis method, they may be costly. Guidelines are the easiest to use but are the least accurate. Numerical methods may be a suitable alternative, but questions remain as to their accuracy in obtaining force distribution on dams and spillways. In order to aid engineers in deciding which method to use, this study compares two physical models with their respective numerical results.

# Description

Two physical models, A and B shown in Figs. 1 and 2, respectively, were fabricated using plexiglas and tested at the Utah Water Research Laboratory (UWRL) in Logan, Utah. The models had the distinctive crest shape characteristic of ogee spillways and included a tangent section. One of the models had a typical flipbucket and the other simply transitioned to a horizontal apron. The models were instrumented with multiple pressure taps in the center of the model that were used to obtain the pressures on the spillway. The models were approximately 1.83 m wide and ap-



**Fig. 1.** Profile of Shape A showing the location of pressure taps and headwater and tailwater elevations

proximately 0.80 m high. The  $P/H_d$  ratio (height of crest/design head) was 2.7 and 6.5, respectively, for Shapes A and B.

#### **Test Facilities**

The models were tested in a flume that was approximately 12 m long, 1.83 m wide, and 1.22 m deep. Flow rates were measured using weight tanks, volumetric tanks, or with an ultrasonic flow meter. Pressures on the spillway were measured using a piezometer board with glass tubes vented to atmosphere. The piezometer board readings provided the average pressure reading at each pressure tap location. Measurements on the piezometer board were readable to within 1.3 mm. Because pressures fluctuated temporally along the crest, most likely due to surface waves, an average pressure was recorded. Pressure fluctuations were directly proportional to the amount of turbulence near the tap. The pressure taps near the crest and down the spillway until submerged by the tailwater had fluctuations on the order of  $\pm 0.5$  cm, whereas pressure taps that were submerged fluctuated on the order of  $\pm 2$  cm.



**Fig. 2.** Profile of Shape B showing the location of pressure taps and headwater and tailwater elevations

Table 1. Model Operating Conditions

	Model A		Model B		
Run	Headwater elevation (cm)	Tailwater <sup>a</sup> elevation (cm)	Headwater elevation (cm)	Tailwater elevation (cm)	
1	86.0	21.4	89.8	52.3	
2	95.7	46.5	95.7	60.4	
3	106.3	70.2	103.6	70.9	
4	109.5	71.7	110.9	80.8	
5	115.7	72.2	—	—	

<sup>a</sup>Referenced from base of each spillway.

#### Model Operation

The models were operated at various upstream total head (piezometric plus velocity head) to design head  $(H_e/H_d)$  ratios as shown in Figs. 1 and 2. For each  $H_e/H_d$  ratio, a unique tailwater elevation was set using stop logs located downstream from the model. The headwater elevation was measured approximately 2 m upstream from the crest of the spillway and the tailwater elevation was measured approximately 3 m downstream from the crest of the spillway. Both the headwater and tailwater elevations were measured using piezometers. Each of the conditions tested had a significant depth of tailwater on the downstream face of the spillway. One of the conditions tested for Model B had the tailwater elevation exceeding the crest elevation. The conditions tested represent typical events for which dams must be designed. Table 1 gives the operating conditions for each of the models.

## Numerical Methodology

The commercially available CFD program, Flow-3D, which solves the RANS equations using a finite volume method, was used to complete the numerical simulation. The program subdivides the Cartesian computational domain into a grid of hexahedral cells. Each ogee-crested dam and spillway was imported into the flow domain. The program evaluates the location of the flow obstacles by implementing a cell porosity technique called the fractional area/volume obstacle representation of FAVOR method (Hirt 1992). The free water surface was computed using a modified volume-of-fluid method (Hirt and Nicholes 1981). For each cell, the program calculates average values for the flow parameters (pressure, velocity) at discrete times using a staggered grid technique (Versteg and Malalasekera 1996). A two-equation renormalized group theory model as outlined by Yahot and



**Fig. 3.** Comparison of numerical model and physical model relative pressures for Model A

Orszag (1986) and Yahot and Smith (1992) was used for turbulence closure.

Because the same methodology was used in the previous study, complete details outlining the numerical modeling can be found in Savage and Johnson (2001). Significant changes include a lengthening of the downstream section approximately 2.75 m and an increase in the downstream boundary depth thereby creating backwater on the spillway. The increase in computational domain allowed sufficient length for the drowned hydraulic jump to stabilize and move toward uniform flow conditions. As expected, this increased the required computational time.

## Results

The flow rates over the spillway crest and the pressures acting on the crest are used to compare the differences between the physical models and numerical models for flows on submerged spillways. Table 2 shows the physical model measured flows and the calculated flows from the numerical model.

The pressure heads  $(H_p)$  on the spillways evaluated have been nondimensionalized by dividing them by the design head  $(H_d)$  for each spillway. The pressure position on the spillway is shown nondimensionally as  $X/H_d$ , with X being the horizontal distance from the crest axis. Fig. 3 provides a comparison of average spillway pressures for five different conditions on Model A;  $0.17H_d$ ,  $0.50H_d$ ,  $0.86H_d$ ,  $0.97H_d$ , and  $1.18H_d$ . Fig. 4 provides a comparison of average spillway pressures for four different conditions on Model B;  $0.90H_d$ ,  $1.39H_d$ ,  $2.05H_d$ , and  $2.66H_d$ .

Table 2. Comparison of Observed Flow Rate versus Computed Flow Rate

Run	Model A			Model B		
	Physical (L/s)	Numerical (L/s)	Percent difference	Physical (L/s)	Numerical (L/s)	Percent difference
1	22.8	22.4	1.8	34.5	34.3	0.6
2	46.6	45.3	2.8	83.2	83.5	-0.4
3	87.1	85.6	1.7	101.6	101.2	0.4
4	125.4	124.2	1.0	139.7	140.3	-0.4
5	132.7	131.1	1.2	_	—	_



**Fig. 4.** Comparison of numerical model and physical model relative pressures for Model B

Using the physical models as the basis, Fig. 5 shows the difference, in centimeters of water, between the numerical model and the physical model data at a given  $X/H_d$  position for Model A. Fig. 6 shows the difference, in centimeters of water, between the numerical model and the physical model data at a given  $X/H_d$ position for Model B. For the comparison, absolute error is used instead of a relative error because many of the crest pressures are nearly atmospheric ( $H_p$ =0). At pressures near zero, even a small difference can result in a large relative error. For example, if a pressure difference of 1 cm of water was divided by a reference pressure of 0.5 cm, a relative error of 200% would result even though the absolute difference is relatively small when considering the size of each model tested.

Although there are no prototype pressure data available for comparison to the numerical solution, the writers scaled the data from Model A Run 5 to prototype dimensions and performed a two-dimensional numerical simulation of the prototype for such a comparison. The physical model data were scaled up 28 times and the numerical model was developed and several runs were completed to ensure grid convergence. The results of this comparison are shown on Fig. 7. Pressures from the numerical model compared quite favorably with the scaled physical model data with the exception of Taps 23–28. The greatest absolute deviation be-



**Fig. 5.** Absolute pressure head differences between the numerical model and physical model for Model A



**Fig. 6.** Absolute pressure head differences between the numerical model and physical model for Model B

tween the numerical simulation and the scaled physical model data for these taps was 1.9 m and it occurred at Tap 26. The relative error of this deviation was 7.8%.

It is interesting to note the significant differences between the numerical Model A Run 5 results and numerical simulation of the prototype structure in the vicinity of Taps 23–28. When numerically modeling the physical scale model, the results are considerably closer to those obtained by modeling the prototype and scaling the physical model to the prototype. It has long been accepted that physical scale modeling provides the baseline for evaluating prototype structures and assessing their performance so it is assumed that the physical model results scale directly to the prototype. However, in the case of the numerical model, the deviations shown demonstrate that more research is required to answer the question as to how well prototype structures can be numerically modeled and to what extent they may be applied. For this reason, the writers are continuing their work in this area until greater understanding is obtained that can be concisely presented.

# Discussion



The flow rate results from Table 2 show that the numerical model provided a reasonable solution, even when the dam crest becomes

**Fig. 7.** Comparison of physical model data scaled to prototype dimensions and numerical simulation of prototype

submerged, as in the case of Model B, Run 4. The flow rates predicted by the numerical model are all within 3% of the numerical model's results for each model investigated.

The data presented in Figs. 5 and 6 demonstrate that numerical modeling is capable of reasonably predicting pressures on spillways with significant tailwater. The concern of modeling supercritical flow transitioning to subcritical flow has been and continues to be a difficult problem to solve, however numerical advances are rapidly reducing the inherent difficulties of this problem. For additional information on the difficulties in numerically modeling supercritical flow, the reader is referred to Rahman and Chaudry (1997), Krüger et al. (1998), and Causon et al. (1999).

Examination of Figs. 5 and 6 show that at worst, 2.5 cm of water of difference between the numerical and physical models. This difference was most pronounced on the vertical face of the dams and is likely due to the increased acceleration the flow experiences as it approaches the crest of the dam. In the submerged zone of flow, the differences on Model A were usually less than 2 cm of water; the majority of differences were less than 1 cm. For Model B the results were similar however the differences in the submerged zone were usually less than 0.5 cm of water. In both model cases investigated, the numerical model was robust enough to provide data that is quite accurate.

Although numerical tools still have limitations (including turbulence representation, aeration and bulking, grid resolution, run times, and numerical instabilities to name a few), there are many areas where current numerical methods may offer increased accuracy over design monographs and be sufficiently accurate for the required application. Numerical models can provide more detail about velocity and pressure distributions than can a physical model and may be more economical in some cases. In the past, engineers have had to rely on model studies and design monographs to obtain data necessary for analysis. For uncontrolled spillways with a relatively simple geometry that operate without flow separation around piers, the writers believe that a numerical model may be sufficient to rapidly obtain information necessary to complete a dam stability analysis.

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