

NUMERICAL SIMULATIONS OF SLOSHING IN RECTANGULAR TANKS

Samuel R. Ransau

Department of Marine Technology
Norwegian University of Science and Technology
Trondheim, Norway
Email: sransau@marin.ntnu.no

Ernst W. M. Hansen*

Complex Flow Design AS
Trondheim, Norway
Email: ernt.hansen@complex-flow.com

ABSTRACT

Simulations of two- and three-dimensional sloshing in rectangular tanks are performed using the commercial CFD code FLOW3D. Small amplitude freely oscillating sloshing and non-linear sloshing due to forced excitation were investigated. The results are compared to both experimental results and other numerical results; and tests are made with different grids. The purpose of this study was the validation of the new VOF algorithm under development at Flow Science and implemented in FLOW3D.

INTRODUCTION

Sloshing of liquids in tanks is a very important type of problems from an engineering point of view, and many studies, using different types of methods have presented in the literature. Analytical, experimental and numerical methods have been used in the past to predict such flows. Rognebakke [1] and Solaas [2] give very good literature surveys of the different methods used for solving sloshing problems. For the marine hydrodynamics and offshore industry, it is crucial to be able to predict such fluid motions since liquid impact can cause critical damage on, or failure of tank structure during violent sloshing. This problem is therefore very important when designing oil tankers, FPSOs and LNG carriers for example [3].

In the present study, a commercial CFD code, FLOW3D, is used to simulate both small amplitude freely oscillating sloshing and nonlinear sloshing caused by forced excitation. The liquid used is water. The global fluid motion is of interest as well as the wave

elevation in some local areas.

We start by describing very briefly the CFD code FLOW3D. We then study three-dimensional small amplitude, freely oscillating sloshing for various rectangular tanks. The tanks are geometrically similar (i.e. the width and depth ratios are kept constant) but their dimensions change. Here we focus the analysis on the influence of the dimensions of the tanks on the period of oscillation of the waves and on the energy loss in the waves. Then two-dimensional non-linear sloshing due to a forced oscillating surge motion of the tank is investigated. The influence of the oscillation frequency, as well as the influence of the initial water depth in the tanks on the wave elevation is studied. Finally three-dimensional non-linear sloshing is also investigated. Here a tank with a square base is forced to oscillate in the longitudinal and diagonal directions. Various oscillation frequencies (close to the natural frequency of the tank) and various initial water depths (not infinitely small) are investigated. The wave elevation as a function of time at different locations in the tank is the results of main concern.

NUMERICAL METHOD

The present study is performed using the commercial general purpose CFD software FLOW3D developed by Flow Science. FLOW3D is based on the finite volume for the spatial discretisation of the Navier-Stokes equations and can simulate a variety of fluid flow. For more details see [4]. In this study, water is used for the liquid and an incompressible fluid model is employed. The evolution of the free surface is determined using a Volume Of Fluid (VOF) method. The VOF method was ini-

*Address all correspondence to this author.

tially introduced by in 1981 by Hirt and Nichols [5]. In the VOF method, a function F , called the fluid fraction function and whose value is one at any point occupied by fluid and zero otherwise, is introduced. The average value of F in a cell represents therefore the fractional value of the cell filled by the fluid. The evolution of the values of F is found by solving the following advection equation (see Ransau [6] for a general description of the VOF method):

$$\frac{\partial F}{\partial t} + \nabla \cdot (\mathbf{u}F) = 0 \quad (1)$$

where \mathbf{u} is the fluid velocity.

In the present study, most of the computations were performed using the last VOF method implemented in FLOW3D and called IFVOF6, and at the time this study was performed, this algorithm was still under development. IFVOF4 and IFVOF5 have also been used for some of the computations. The aim was to evaluate the improvement made in IFVOF6 as it is an extension of both IFVOF4 and IFVOF5. All three methods solve the advection equation (1) (see the Flow Science technical notes [4] for more details).

NUMERICAL EXAMPLES

In this section we will present a few numerical examples of sloshing in rectangular tanks and try to evaluate in what kind of sloshing problems can be simulated with good confidence with FLOW3D.

Three-dimensional small amplitude sloshing

The first case studied is a three-dimensional small amplitude, freely oscillating sloshing. Keulegan [7] has performed experiments on several tanks which have different dimensions but where width and depth ratios are kept constant. In this case, the quantities that we will compare to Keulegan's results in [7] are the period of oscillation and the energy loss in the wave, measured by a modulus of decay. The latter is a quantitative estimation of the energy dissipation in standing waves and contains two main contributions: energy loss due to boundary layer effects and energy loss due to interfacial effects. In addition, we check the volume error since we know that mass conservation is usually a critical issue for CFD softwares. The results are presented in tables 1 and 2. T is the period of oscillation and is given in seconds. Further, Vol_err is the cumulative volume error with respect to the initial volume and given in percent. Finally, α is the modulus of decay as defined in [7]. The dimensions of each basin is given in table 1 as well as the number of cells used for each simulation. The grids used hexahedral cells where the 6 faces are rectangles. The grid is clustered towards the free surface and the maximum aspect ratio is 3.

Basin: dim (cm)	Grid	T (s) Exp	T (s)
A: 242x52.6x103	(121x26x52)	1.89	1.91
B: 94.7x20.6x40.4	(94x20x42)	1.2	1.19
C: 47.4x10.1x20.1	(48x10x21)	0.83	0.85
D: 37x8x15.7	(60x10x60)	0.73	0.74
E: 27.8x6.1x11.8	(28x7x14)	0.64	0.61
F: 23.8x5.2x10.1	(48x10x21)	0.59	0.63
D2: 37x8x15.7	(30x10x60)	0.73	0.74
D3: 37x8x15.7	(120x10x60)	0.73	0.74
D4: 37x8x15.7	(240x10x60)	0.73	0.75

Table 1. Comparison of the experimental and numerical periods of oscillations.

Basin	α Exp	α	VolLerr (%)
A	0.0057	0.0046	8.41e-3
B	0.0116	0.0119	1.18e-2
C	0.0197	0.0396	-2.24e-3
D	0.0234	0.0182	5.45e-3
E	0.0228	0.0336	5.32e-4
F	0.0325	0.0372	3.33e-3
D2	0.0234	0.0384	8.71e-3
D3	0.0234	0.0280	1.25e-2
D4	0.0234	0.0207	3.04e-2

Table 2. Comparison of the experimental and numerical modulus of decay; and cumulative volume error after 50s.

Tables 1 and 2 shows that the results obtained with FLOW3D are in good agreement with the experimental results presented by Keulegan [7]. In addition, the cumulative volume error presented in table 2 shows that volume conservation is not an issue for this kind of problem. Figure 1 shows the evolution of the water elevation at the centerline of the walls where x has the minimum value. As expected, since the tank is not excited, the wave elevation is decreasing and will be completely damped out after some time. Figure 2 shows a two-dimensional view of the pressure field (in the plane x - z at the middle value of y) close to the free surface at time $t=T$. Figure 3 shows the pressure field in the plane y - z close to the free surface at different x -locations at time $t=T$. It can clearly be seen from figure 3, that there is no

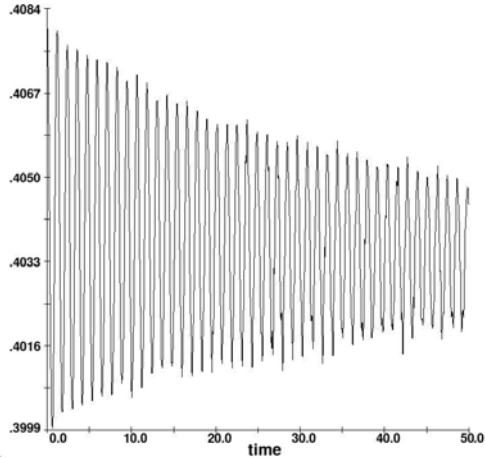


Figure 1. Wave elevation at the wall where x has its minimum value for the basin B.

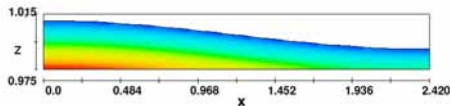


Figure 2. Two-dimensional view of the pressure field. x - z plane at the middle value of y and at time $t=T$ for the basin A.

three-dimensional (in the y -direction) effects in this flow. It is a two-dimensional flow. Figure 4 shows the fraction of fluid close to the free surface in the x - z plane passing through the center of the tank at different time instants. For this kind of cases (small amplitude freely oscillating sloshing), FLOW3D gives very good results compared with experiments and may be used with good confidence for this kind of problems.

Nonlinear two-dimensional sloshing

In this example, we simulate violent sloshing in a partially filled tank forced to oscillate in a frequency domain close to its lowest natural frequency. The tank is only excited in surge (horizontal direction). The tank dimensions in the x -, y - and z -directions are 1.73m, 0.2m and 1.05m respectively. The amplitude of oscillation was 0.03m. The water depths h used were 0.5m and 0.6m. At each time step, the water elevation was recorded at the position WP as shown in figure 5. In the experiments conducted by Faltinsen et al. [8], it was observed that the water elevation did not vary in the y -direction. We will there performed most of the simulations for this case with a quasi two-dimensional grid having only one cell in the y -direction. However, one three-dimensional simulation is also run in order to check that no three-dimensional effects appear using FLOW3D. The grids used hexahedral cells where the 6 faces are rectangles. In the first case simulated in the present study, the frequency of

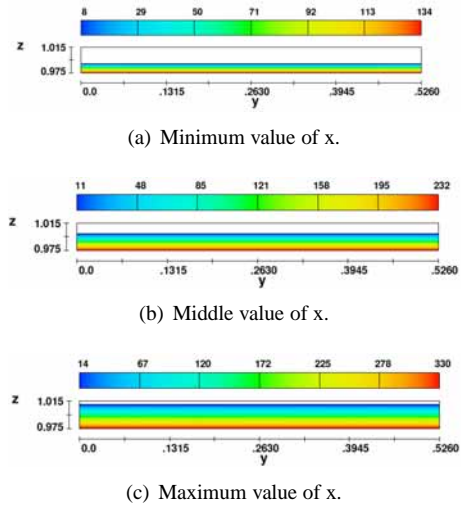


Figure 3. Two-dimensional views of the pressure field close to the water surface in the y - z plane after half a period for the basin A.

the surge oscillation was $T=1.5$ s, and the water depth in the tank was 0.6m. The results are presented in fig. 6. For this simulation a regular grid with 125760 cells was used. In [8], both experimental and numerical results were presented. The numerical results come from a modal analysis solving the potential flow equations. For this case, the results given by FLOW3D compare very well with the experimental results presented in [8]. They compare better than the modal analysis results presented in [8]. This is in particular true for the rate of decay of the wave elevation. Figure 6(b) shows the cumulative volume error. It is 0.58% after 50 seconds, which is a relatively low value.

We then simulate the same case with a slightly different grid. In the new grid, the fluid domain was divided in two parts: a lower and an upper part. The lower part goes from the bottom of the tank to 0.2m below the water surface at rest. The upper part goes from this line to the tank ceiling. In the upper part the grid is the same as in the previous simulation, while in the lower part it is coarser. The total number of cells is now 94320. The aim in doing was to know how the refinement of the grid close to the tank bottom influenced the results since the water height was relatively small. The results are presented in fig. 7. The differences between figs. 6 and 7 are so small that we can say that coarsening the grid at the tank bottom has no effect on the accuracy of the results.

Further, we performed two three-dimensional simulations of this case. We used the same two grids as previously but increase the number of cells in the y -direction from 1 to 20. The total number of cells in those three-dimensional grids was 922240 and 691680. The results are presented in figs. 8 and 9. The analysis of these figures and of a three-dimensional view of the flow field confirms that there is no variation of the wave elevation in the y -

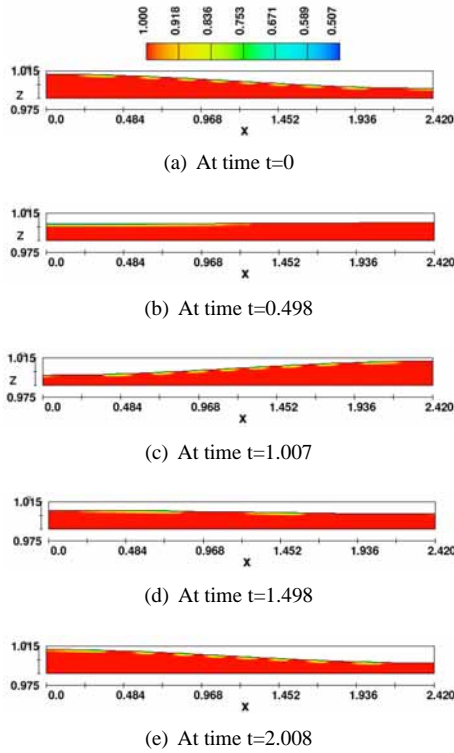


Figure 4. Two-dimensional views of the fraction of fluid close to the water surface for the basin A at different time instants.

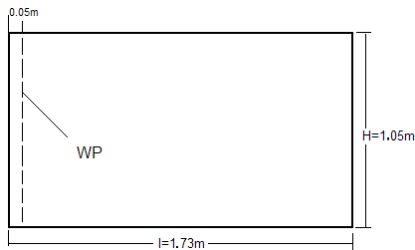
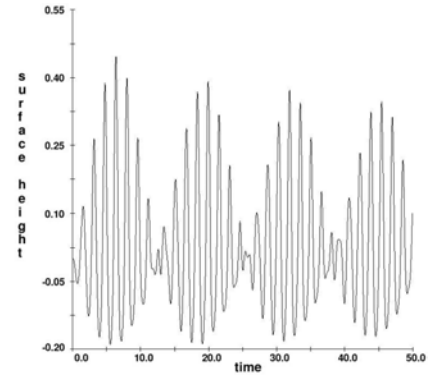


Figure 5. Definition of the tank and of the wave probe.

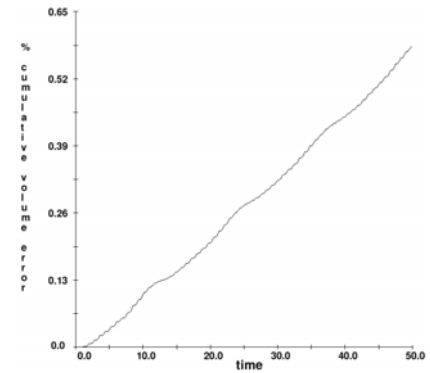
direction. Furthermore, after 50s, the cumulative volume error in the cases where the non regular 2D, regular 3D and non regular 3D grids are used is 0.59%, 0.6% and 0.6% respectively.

Further, the simulation presented in fig. 6 was also simulated with two other VOF algorithms implemented in FLOW3D, IFVOF=4 and IFVOF=5. The results are presented in table 3 and fig. 10 and it is clear that with IFVOF6 the accuracy of the mass loss is highly decreased for this type of problems.

Next we simulated a case similar to the one reported in fig. 6 but this time with a period of oscillation T equal to 1.3s. The results are presented in fig. 11 and comparing to those reported in [8], it can be observed that the maximum wave elevation is



(a) Wave elevation.



(b) Cumulative volume error.

Figure 6. Computed free surface elevation (in m) and cumulative volume error (in%) for $h=0.6m$ and $T=1.5s$ with a 2D regular grid.

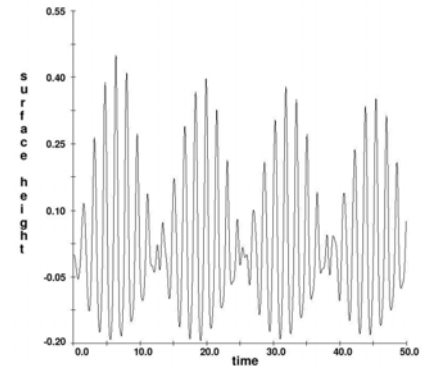


Figure 7. Same as in fig. 6 but with a 2D non regular grid.

	IFVOF4	IFVOF5	IFVOF6
Vol_err (%)	-5.23	5.68	0.58

Table 3. Comparison of the cumulative volume error in percent after $t=50s$ with IFVOF4, IFVOF5 and IFVOF6.

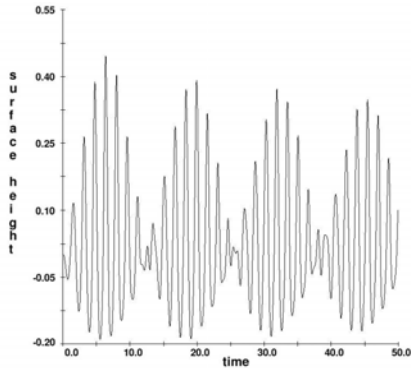


Figure 8. Same as in fig. 6 but with a 3D regular grid.

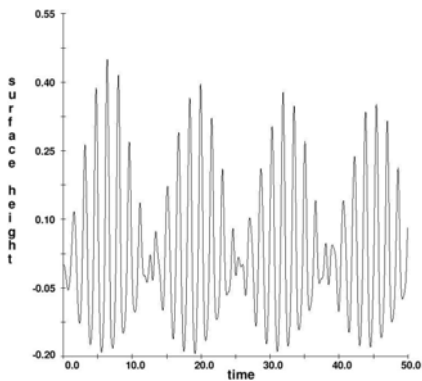


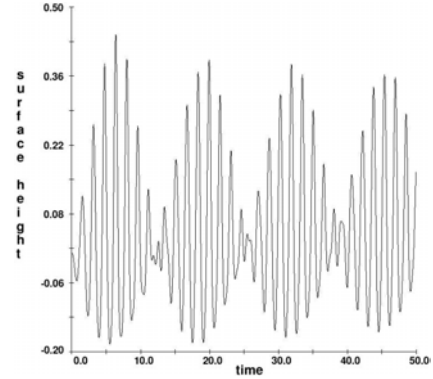
Figure 9. Same as in fig. 6 but with a 3D non regular grid.

20 to 30% larger in our simulation than in the experiments. In addition, the cumulative volume error after 50s is 0.39%.

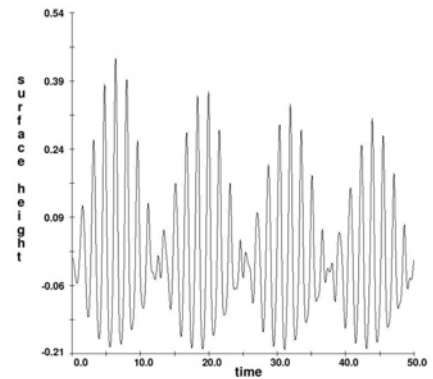
Next we simulated a case similar to the one reported in fig. 6 but this time with the initial water depth and the period of oscillation were equal to 0.5m and 1.875s, respectively. The results are presented in fig. 12 and compare very well with those reported in [8] and the cumulative volume error after 50s is 0.53%.

At last we simulated a case similar to the one reported in fig. 6 but this time with the initial water depth and the period of oscillation were equal to 0.5m and 1.4s respectively. The results are presented in fig. 13 and agree well with the numerical presented in [8], but there exist larger difference between our results and the experimental ones. The cumulative volume error in the present simulation is 0.38%.

Based on the results presented in figs. 6 to 13, it can be concluded that generally FLOW3D gave us good results for this type two-dimensional nonlinear sloshing.



(a) Wave elevation.



(b) Cumulative volume error.

Figure 10. Same as in fig. 6 but with IFVOF4 and IFVOF5.

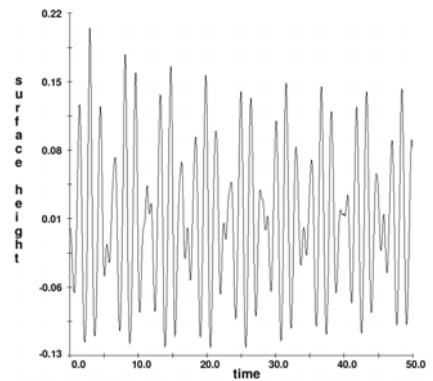


Figure 11. Same as in fig. 6 but with $T=1.3s$.

Nonlinear three-dimensional sloshing

In this example we simulated three-dimensional flow in the tank which is forced to oscillate in surge with oscillation frequencies σ close to its lowest natural frequency. The tank had a square base with width and breadth equal to 0.59m and a height equal to 0.8m. The water depths and amplitude of oscillation used here were made dimensionless using the tank width. The

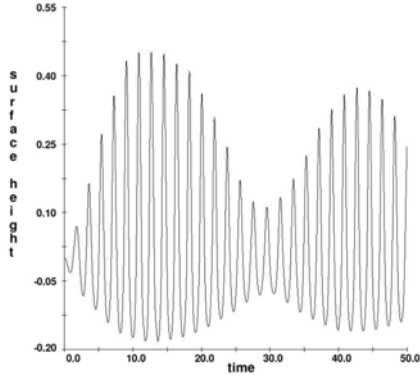


Figure 12. Same as in fig. 6 but with $h=0.5\text{m}$ and $T=1.875\text{s}$.

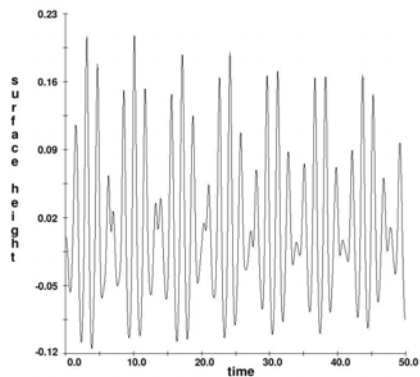


Figure 13. Same as in fig. 6 but with $h=0.5\text{m}$ and $T=1.4\text{s}$.

dimensionless water depths studied were $h=0.34$ and $h=0.508$. The amplitude of oscillation was approximately 0.46cm which gives a dimensionless amplitude of oscillation H equal to 0.0078 . During the simulation, the wave elevation was recorded at different positions in the tank. Those positions were called WP1, WP2, WP4, WP6 and are shown in fig. 14. WP1 and WP6 are located 40mm away from the tank wall while WP2 and WP4 are located on the tank wall.

In the following simulations the tank is forced to oscillate longitudinally along the x -axis or diagonally in the xy -plane. The lowest natural frequency for this tank, σ_1 , is given in [9] and is equal to 5.3278 . As Faltinsen et al. [9] mentioned that the flow was turbulent, the renormalized group model (RNG) was used for turbulence modelling purposes in the present study.

In the first case that was simulated, the tank was forced to oscillate longitudinally along the x -axis. The dimensionless water depth h was equal to 0.34 and the ratio σ/σ_1 was equal to 0.98 . The wave elevation at WP1 and WP6 is presented in fig. 15. The amplitude of the waves obtained by using FLOW3D is ten times smaller than the amplitude observed by Faltinsen et

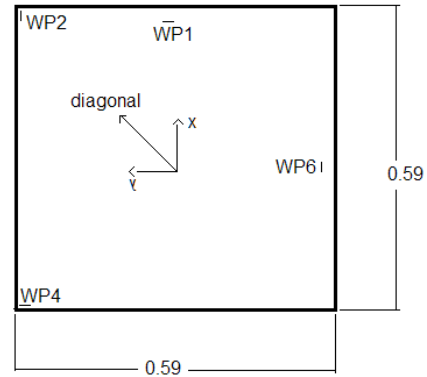


Figure 14. Bird view of the tank with the positions of the wave probes.

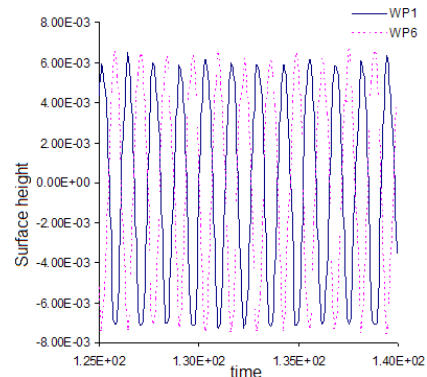


Figure 15. Computed free surface elevation (in m) at wave probes WP1 and WP6 for longitudinal excitation with for $h=0.34$ and $\sigma/\sigma_1 = 0.98$.

al. [9] in their experiments. We have some difficulties explaining this large difference especially since the mass loss is not very important: the cumulative volume error is 0.19% after 140s .

In the next case which was simulated, the tank was forced to oscillate diagonally in the xy -plane. The dimensionless water depth h was set equal to 0.508 and σ/σ_1 was set equal to 0.9495 . The wave elevation is presented in fig. 16. The oscillations obtained with FLOW3D are also in this case much smaller than those presented in [9]. Once more it is difficult to explain such a big difference.

Finally, in the last case which was simulated, the tank was forced to oscillate diagonally in the xy -plane. The dimensionless water depth h was set equal to 0.508 and σ/σ_1 was set equal to 1.1 . The results are presented in fig. 17. First it can be observed that the wave elevations obtained using FLOW3D at wave probes WP1 and WP6 are 35% smaller than those reported in [9]. In addition, the results presented by Faltinsen et al. [9] at WP1 and WP6 contains a main oscillation with frequency equal to about

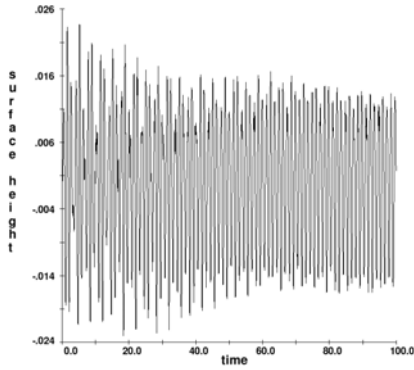


Figure 16. Computed free surface elevation (in m) at wave probes WP2 for diagonal excitation with $h=0.508$ and $\sigma/\sigma_1 = 0.9495$.

0.83s and a secondary oscillation with frequency equal to about 8s; and two packs containing 10 oscillations each are observed. In the results obtained with FLOW3D the main frequency of oscillation of the wave elevation at WP1 and WP6 is about 1s. However, the secondary frequency is about 5s. Here 4 packs containing 5-6 oscillations each were observed. Further, the cumulative volume error was equal to 0.114% after 25s.

CONCLUDING REMARKS

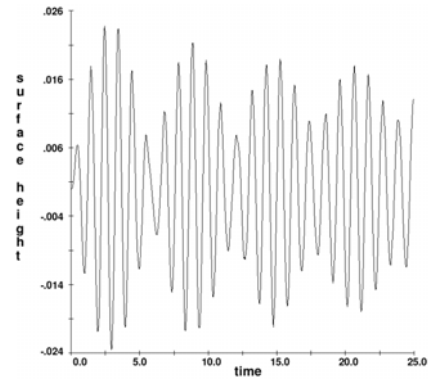
Numerical simulations of sloshing in rectangular tanks were performed using the general CFD tool FLOW3D. The cases studied were three-dimensional small amplitude sloshing, two dimensional nonlinear sloshing and finally three-dimensional nonlinear sloshing. For the most of cases studied in the first two types of problems FLOW3D gave very good results compared with experimental results. It was also observed that the new VOF algorithm implemented in FLOW3D called IFVOF6 gives much better results than the IFVOF4 and IFVOF5 methods. The mass loss after a simulation time approximately equal to 30 periods of oscillations was less than 1% with IFVOF6. Keeping in mind that mass conservation is usually a critical issue in CFD simulations, it can be concluded that FLOW3D (and IFVOF6) is a good tool for simulating this kind of problems.

As for three-dimensional violent type of sloshing, the results obtained with FLOW3D were more questionable. However, numerical computations of such violent three-dimensional flows are still a big issue for general purpose CFD tools. In addition, this study was performed using a version of FLOW3D where IFVOF6 was under the development.

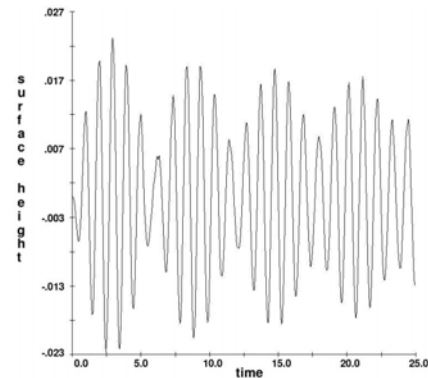
REFERENCES

- [1] Rognebakke, O. F. 2002 *Sloshing in rectangular tanks and interaction with ship motions*. Dr.Ing. Thesis 2002-16, Norwegian University of Science and Technology, Trondheim, Norway.

Figure 17. Computed free surface elevation (in m) at wave probes WP1 and WP6 for diagonal excitation with $h=0.508$ and $\sigma/\sigma_1 = 1.1$.



(a) Wave elevation at wave probe WP1.



(b) Wave elevation at wave probe WP6.

- [2] Solaas, F. 1995 *Analytical and numerical studies of sloshing in tanks*. Dr.Ing. Thesis 1995-, Norwegian University of Science and Technology, Trondheim, Norway.
- [3] Valsgrd, S. and Tveitnes, T. 2003 *LNG Technological Developments and Innovations Challenges with Sloshing Model Testing*. Det Norske Veritas AS Paper Series, No.2003-P005.
- [4] *Flow Sciences website, general description of FLOW3D*: <http://www.flow3d.com/flow3d.htm> and *technical note bibliography describing the methods more in details*: <http://www.flow3d.com/Bibliography/technote.htm>.
- [5] Hirt, C. W. and Nichols, B. D. 1981 *Volume Of Fluid (VOF) method for the dynamics of free boundaries*. J. Comp. Physics, vol. 39, pp. 201-225.
- [6] Ransau, S. R. 2004 *Numerical methods for flows with evolving interfaces*. Dr.Ing. thesis 2004:12, Norwegian University of Science and Technology, Trondheim, Norway.
- [7] Keulegan, G. H. 1958 *Energy dissipation in standing waves*

- in rectangular basin*. Journal of Fluid Mechanics, vol. 5, pp. 33-50.
- [8] Faltinsen, O. M., Rognebakke, O. F., Lukovsky, I. A. and Timoka, A. N. 2000 *Multidimensional modal analysis of nonlinear sloshing in a rectangular tank with finite water depth*. Journal of Fluid Mechanics, vol. 407, pp. 201-234.
- [9] Faltinsen, O. M., Rognebakke and Timoka, A. N. 2003 *Resonant three-dimensional nonlinear sloshing in a square-base basin*. Journal of Fluid Mechanics, vol. 487, pp. 1-42.