DEVELOPMENT OF A NUMERICAL APPROACH FOR SIMULATION OF SAND BLOWING AND CORE FORMATION

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Abstract

In the present paper, a three-dimensional and transient numerical approach was developed to simulate sand blowing and core formation based on the fundamental laws of mass and momentum conservation. The flowing sand is treated as a continuous phase and tracked by solving a scalar convection equation supplemented by drifting and lifting models. The completely packed sand is treated as a non-flowing scalar concentration that become a porous medium through which the continuous carrier fluid can flow. A screening scheme was developed to block sand, but let fluid pass through venting areas. The above mentioned scalar equations and fluid mass and momentum conservation equations are solved for using a finite-volume method in a fixed-grid of cells. Sand blowing into a core box with the insertion of a "H" shape was simulated to illustrate the new model.

Introduction

Modeling of sand-air two-phase flow (sand-core blowing) has significant importance in helping to obtain optimum molds. Usually two methods could be considered to simulate this kind of flow. One is the Eulerian-Eulerian (or two-fluid) method and the other is Eulerian-Lagrangian method. In the former method, the particulate phase is considered to be a continuous fluid interpenetrating and interacting with the continuous fluid phase (Gidaspow, 1994). One set of conservation equations for each phase plus some emperical correlations for interface transport are solved to simulate the concerned flows. In the latter method, the fluid phase is solved in an Eulerian grid while the particulate phase is tracked using the Lagrangian method by representing it through a number of computational particles (called parcels) each of which represents a group of particles with identical properties (Dukowicz, 1980). In this method, the particle volume fraction is usually neglected. To account for its effects, the particle phase is considered to be both a continuous and a discrete phase. For example, when interparticle stresses are needed, the particles are treated as a continuous phase (Patankar and Joseph, 2001). Even though both methods have some advantages, they are computationally expensive for the simulation of practical problems.

In this paper, we present a simple approximate method to simulate sand-core blowing and core formation. Specifically, the fluid flow is simulated by solving a mixture mass and momentum conservation equation. Sand is treated as a continuous phase and tracked by solving a scalar convection equation supplemented by drifting and lifting models. In the following section, the model equations are described. Then, a sample problem is presented to demonstrate the application of the developed method, followed by a brief conclusion.

Governing Equations and Numerical Method

The fluid field is represented by the following mass and momentum conservation equations:

$$\nabla \cdot \mathbf{U} = 0 \tag{1}$$

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} = -\frac{1}{\rho} \nabla P + \frac{\mu}{\rho} \nabla^2 \mathbf{U} - K\mathbf{U} \tag{2}$$

where K is the drag coefficient for fluid flow through a porous medium, is defined by

$$K = \frac{\alpha \nu (1-\epsilon)^2}{\epsilon^2 d_p^2} [1 + \frac{\beta}{\alpha(1-\epsilon)} Re] \tag{3}$$

The values of constants α and β are 180.0 and 3.0 respectively while the Reynolds number is defined by

$$Re = \frac{\rho d_p |\mathbf{U}/\epsilon|}{\mu_c} \tag{4}$$

The suspended sand particles are tracked by solving the following scalar equation for the sand concentration

$$\frac{\partial C_s}{\partial t} + \nabla \cdot (\mathbf{U} C_s) = \nabla \cdot (\mathcal{D} \nabla C_s) \tag{5}$$

For convenience in the following description, this scalar is named “Scalar1”. If the concentration of sand particles reaches the critical value (packing limit), they are packed (i.e.,converted from the moving to the non-moving scalar field). The packed sand is treated as a porous medium through which fluid can flow and is represented as a stationary scalar, named "Scalar2", again for the convenience of description. The packed sand can be resuspended into the fluid if the shear stress at the packed bed-fluid interface is large enough. Resuspending of the packed sand is simulated by a lifting flux which is calculated according to the equations presented in [2] (Brethour, 2001). To account for relative movement between sand and air, the relative velocity between them is calculated by:

$$\mathbf{V}_r = \frac{\rho_p - \rho_c}{\rho} \alpha_c \frac{r_p}{\frac{3}{8}\rho_c C_D |\mathbf{V}_r|} \nabla \cdot P \tag{6}$$

This equation was derived by simplifying the two-fluid momentum conservation equations for sand particles and carrier fluid with drag coefficient C_D calculated by some empirical correlations. The following two correlations are possible choices. One was derived by Ishii et al(1979), which is:

$$C_D = \begin{cases} \frac{24}{Re_m}(1 + 0.1 Re_m^{0.75}) & \text{if } Re_m \leq 1000 \\ 0.45 \left[\frac{1+17.67(\sqrt{1-\alpha_p}\frac{\mu_c}{\mu_m})^{\frac{6}{7}}}{18.67\sqrt{1-\alpha_p}\frac{\mu_c}{\mu_m}} \right]^2 & \text{if } Re_m > 1000 \end{cases} \tag{7}$$

where

$$Re_m = \frac{\rho_c |\mathbf{V}_r| d_p}{\mu_m} \tag{8}$$

and

$$\mu_m = \mu_c \left(1 - \frac{\alpha_p}{\alpha_{pm}}\right)^{-2.5\alpha_{pm}} \tag{9}$$

The other was derived by Ergun (1959) and was suggested by Gidaspow (1994) to be valid for $\alpha_c < 0.8$. It can be written in the following form:

$$C_D = \frac{24}{Re_r}(8.33\frac{\alpha_p}{\alpha_c} + 0.0972 Re_r) \tag{10}$$

where

$$Re_r = \frac{\rho_c d_p |\mathbf{V}_r|}{\mu_c} \tag{11}$$

The former correlation includes effects of particle volume fraction through the calculation of a mixture viscosity while the latter one includes this effect directly in the expression for C_D. Based on some test calculations, it was demonstrated that the correlation represented by Eqn.(10) is more robust and should be used in Eqn.(6).

Equations (1-2) and (5) are recast to include the fractional areas and volumes of control elements that are open to fluid. This fractional-area-volume-obstacle-representation ($FAVOR^{\text{TM}}$) method is used to handle complicated geometries in a fixed-grid of cells. Detailed information can be found in [6] (Hirt and Sicilian, 1985). To solve for a relative velocity between sand and air, a total relative velocity in the direction of the pressure gradient according to Eqn.(6) is solved for. Then this total relative velocity is partitioned into x, y, and z components based on the components of the pressure gradient. By doing this, the components of relative velocity are easily obtained.

In sand-core blowing systems, sand approaching venting holes is blocked there by screens. Therefore, venting holes are impenetrable to sand and penetrable to air. To simulate this process, the "Scalar1" attempting to pass through a vent is transfered into the packed sand

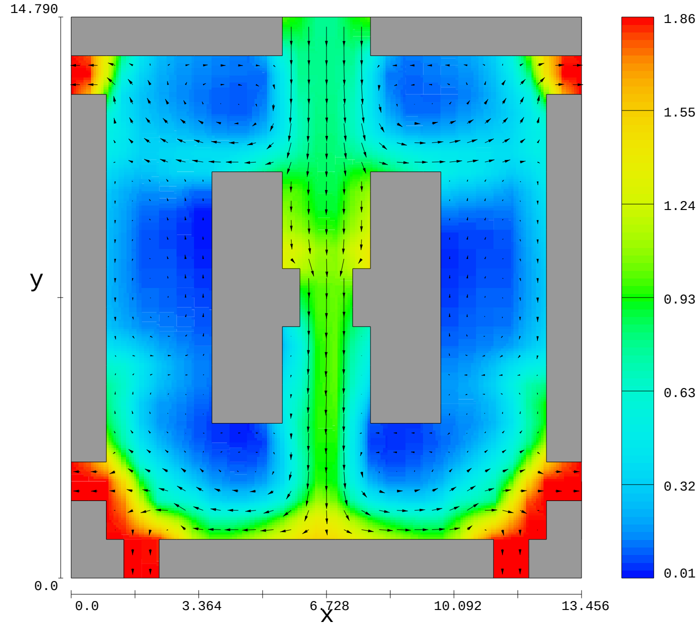

Figure 1: Sand Concentration Distribution at Time=0.2s

"Scalar2" (called "screening") in the cells immediately adjacent to the boundary representing the venting holes. If sand particles are impacting on a solid wall, we currently let them deposit there. To account for this physical process, "Scalar1" is transfered into "Scalar2" (called "depositing") in the cells immediately adjacent to solid walls. The deposited amount of "Scalar1" is calculated based on the normal (to the wall) component of velocity at the cell center in question.

The developed model was implemented in *FLOW-3D*®—a commercial CFD code. A sample simulation is provided in the following section.

A Sample Problem

To demonstrate the application of the numerical approach described in the previous section, the sand blowing into a square box with the insertion of a "H" shape is simulated. The same geometry was also used for their simulation of sand core blowing by Bakhtiyarov and Overfelt (2001). Sand is blowing into the box through the top and outlet vents are placed in the other three sides. Figures 1 through 3 are presented to illustrate the sand core formation process by showing sand concentration inside the box and velocity vectors at different times. Sand particle diameter used in the simulation is 100 μm while inlet sand concentration, air velocity, and the densities of sand and packed sand bed are 1.0 g/cm^3, 300 cm/s, 2.65 g/cm^3 and 1.86 g/cm^3 respectively. The computational model of the type described here is based on a variety of averages. It is very difficult in a practical computational tool to consider detailed particle dynamics. The interactions between particles, e.g., collisions, collective effects, one particles' wake affecting other particles, etc., can not be modeled separately. It is essential to use a model that describes the average behavior of many particles. Although multiple particle sizes could be included in our model by defining more scalar variables to record the concentrations of those sizes, we do not feel this is justified at this point, because of the lack of data. In particular, there is very little useful data available that can be used to simply validate packing shapes (histories) or provide pressure histories at selected locations

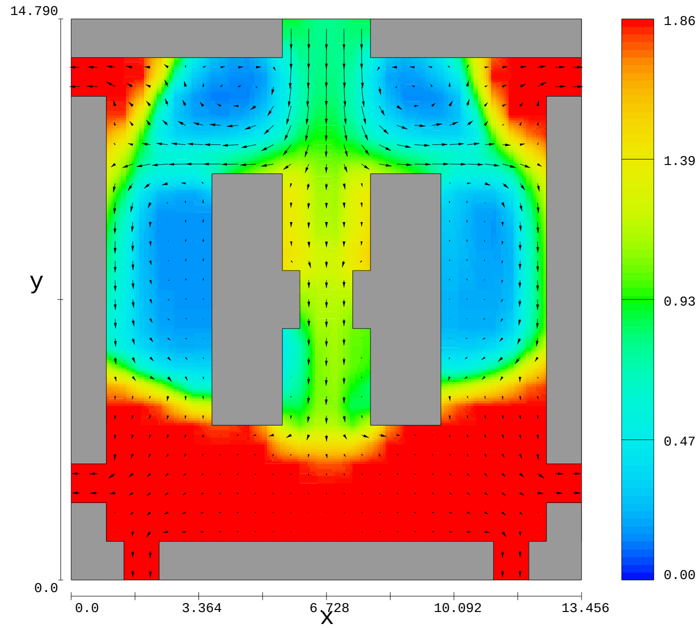

Figure 2: Sand Concentration Distribution at Time=0.4s

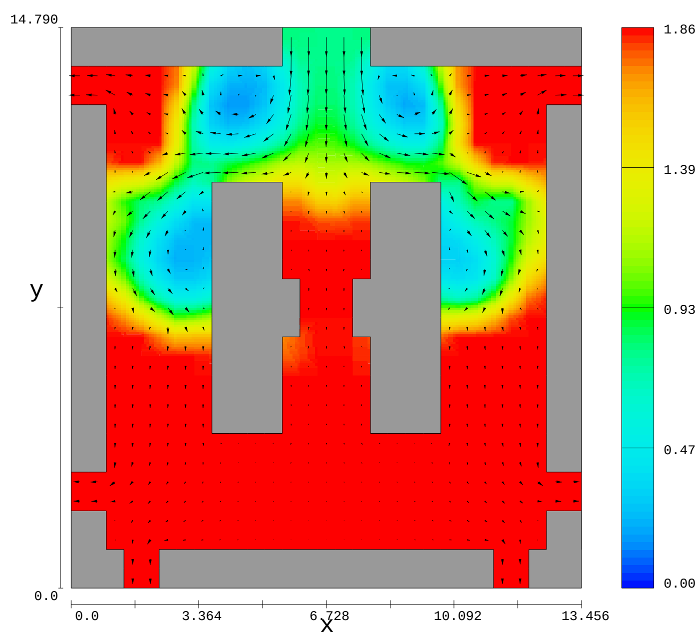

Figure 3: Sand Concentration Distribution at Time=0.5s

in a core box. Almost no data is available for checking predictions of defects in sand cores. Therefore, a representative sand size is used here in the simulation. All vents are exposed to the atmosphere. These figures show that the presented approach can simulate the sand blowing and core formation process. In particular, vents with arbitrary numbers and locations can be conveniently modeled due to the adoption of convecting and stationary scalars. This feature is very useful to simulate sand core formation process in real scenarios where vent number and locations have significant effects on the quality of the final products.

It is always important to make some detailed validation of a new numerical approach by comparing predictions against experimental data or analytical solutions. However, very few proper experimental data on sand blowing and core formation are available in the open literature, and analytical solutions are difficult to get even for the simplest case due to the complexity of the sand core blowing process. To make a check on accuracy, a global mass and momentum balances are carried out. Both global mass and momentum conservation are satisfied. Further validation will be performed when good experimental data are made available.

Conclusions

A numerical approach was developed to simulate the sand blowing and core formation process. Sand is tracked by solving a scalar convection equation with correction of lifting and drifting flux. The drifting flux was obtained by solving a simplified two-fluid momentum conservation equation in the direction of pressure gradient followed by a partition in three coordinate directions respectively. The packed sand is treated as a porous medium. Due to this feature of handling both suspended and packed sand, the present numerical approach has the capability to simulate sand blowing and core formation processes. To demonstrate this, sand blowing into a square box with the insertion of a "H" shape was simulated.

Nomenclature

C_D	dimensionless drag coefficient
C_s	concentration of sand particles
d_p	particle diameter
$\mathcal{D}$	diffusion coefficient
K	drag coefficient
P	pressure
Re	Reynolds number, defined by Eqn.(4)
Re_m	Reynolds number, defined by Eqn.(8)
Re_r	Reynolds number, defined by Eqn.(11)
$\mathbf{U}$	velocity of continuous phase
$\mathbf{V}_r$	relative velocity between sand particles and air

Greek Symbols

α	constant in Eqn.(3)
α_c	volume fraction of air
α_p	volume fraction of sand particles
β	constant in Eqn.(3)
ϵ	porosity

μ dynamic viscosity of mixture
μ_m viscosity defined by Eqn.(9)
ν kinematic viscosity of mixture
ρ density of mixture
ρ_c density of air
ρ_p density of sand particles

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