Numerical simulations have become an integral part in the development of process parameters in the casting industry. But in spite of recent developments in computer technology as well as optimizations of numerical modeling algorithms, running fully coupled CFD analyses still requires substantial amounts of time. Due to a short cycle time, heat transfer is usually omitted from the simulation, and more emphasis is put into a flow pattern design. Considerations of the heat transfer rate and heat flow patterns becomes more important in a thin-wall casting. Due to the high temperature gradient between molten metal and the die cast die steel, flow in boundary layers will be affected by partial solidification.

A simple algorithm was developed to trace temperature distribution in metal during cycle time as well as trace solid-liquid interface. Optimization of a cycle time at slow as well as fast-shot stage is considered.

Examples in this article are solved with the help of MATLAB® functions. All results are verified using the general purpose CFD code Flow3D®.

**Introduction**

Thin-wall pressure die castings have become increasingly important as the design world targets more closely the optimization of mass, properties and weight. In vehicles, this translates to greater miles-per-gallon, lower emissions, less energy to move systems through inertia moments and use of less material for each casting application. The European auto manufacturers (with the U.S. companies following) have made thinner, stronger, stiffer, castings a high priority as the future vehicles are designed. They see this engineering direction as a dominant force in the decade ahead. As castings are lightened, the wall sections must be relied on to deliver the properties expected. Optimized design safety factors have become a goal, adding the least material possible in pursuit of stable application. Engineered alloys, heat treatment (T5, T6) on thin-pressure castings, minimized distortion and wall sections only as thick as needed have emerged as the features on projects in the present, not just requirements of the future. These new trends demand that pressure die casters pay close attention to the heat loss incurred in the metal transfer moments. Optimization of heat transfer in the shot sleeve is one of those opportunities to improve the integrity of the casting sections. For thin castings, it is a critical concern.

Heat transfer analyses can give an important insight into the die casting process. Predictions of the heat flow pattern and temperature distribution of every stage of the die casting process can help in optimizing the overall process, which will result in higher quality castings. Due to the complexity of the shape of the die castings and highly nonlinear nature of the solidification problem, exact solutions are not available except in a few simplified cases. With the development of digital computers, a variety of numerical methods have been used. Solutions achieved by numerical algorithms are found to be in good correlation with actual results. In today's business environment when time from design to market is reduced to a minimum, engineers tend to take a shortcut by omitting heat transfer analyses from the flow analyses. Although it may be warranted in a majority of the cases, process design of the thin-wall die casting requires one to pay close attention to a temperature distribution during cavity fill because early solidification can significantly influence the part quality.

**Description of the Problem**

The typical cold chamber die casting process has several stages (Figure 1). It starts with filling shot cylinder with molten metal (Figure 1a), then the metal is given time to settle down (Figure 1b) before slow shot can start (Figure 1c). Slow shot is designed to prevent air entrapment in the shot cylinder. Calculation of the slow-shot velocity is based on the theory of flow in an open channel. Several velocity profiles are used in process design. The two most common designs are constant velocity profile, where after passing the pour hole, the plunger accelerates to a calculated velocity value, and variable velocity profiles, where the plunger constantly accelerates until the shot sleeve is completely full (Figure 1d). Fast shot starts between positions where the shot sleeve is completely full and before metal reaches the gates. Then, metal fills the cavity of die cast die with the plunger moving at fast-shot velocity.

Heat loss from the molten metal occurs due to conduction on the metal — shot sleeve interface. Heat losses by radiation and convection from the open area of the metal in the shot sleeve are ignored for these analyses. On the fast-shot stage of the die cast process, heat losses occur on liquid metal — cavity steel interface. Optimization of every stage of the die cast process will result in reduction of heat losses.

**Figure 1** – Stages of the die cast process; a) metal is ladled into the shot sleeve; b) metal is settle down; c) plunger start moving; d) shot cylinder filled.

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Governing Equation

We are solving Stefan's problem of solidification. Initially, material is in liquid stage. At time \( t = 0 \), material comes into contact with the steel of the die cast die, which is at temperature below temperature of solidification. As a result, a liquid-solid interface forms and starts moving away from the wall. The temperature distribution is governed by a heat conduction equation.

In a cylindrical coordinate system:

\[
\frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{\partial T}{\partial \theta} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t}
\]  

(1)

In the Cartesian coordinate system:

\[
\frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t}
\]  

(2)

Where \( \alpha = k/\rho C \), \( k \) is thermal conductivity, \( \rho \) is density of metal, and \( C \) is specific heat.

The boundary conditions for this problem are

\[
T(x = 0, t) = T_w \quad \text{and} \quad T(x = l(t), t) = T_m
\]

where \( T_w \) is the mold wall temperature and \( T_m \) is the melt temperature. We need an additional condition that would express the velocity of the moving solid-liquid interface as a function of heat transfer in both the solid and liquid phases. This is called the Stefan condition and can be expressed as

\[
\rho L \frac{dS}{dt} = k \frac{\partial T}{\partial x} + k \frac{\partial T}{\partial x} |
\]

(3)

where \( L \) is the latent heat realized on solidification, and \( S \) is location of solid-liquid interface.

Enthalpy Method

Using the enthalpy method allows one to avoid direct tracking of the solid-liquid interface. The enthalpy formulation eliminates explicit reference to a moving boundary. An enthalpy function \( H(T) \), which is total heat energy per unit mass (i.e., the specific enthalpy) and includes the latent heat \( (L) \) and the sensible heat in each phase, is defined as

\[
H = \begin{cases} 
T_0 \rho \theta & T < T_f \\
H_1 \leq H \leq H_2 \ @ T = T_f \\
T_0 \rho \theta + L \ @ T > T_f 
\end{cases}
\]

(4)

where \( H_1 = T_0 \rho \theta \) and \( H_2 = T_0 \rho \theta + L \) and \( T_f \) is the fusion temperature.

The metal is solid at its melting (fusion) temperature with \( H = H_1 \), and is liquid at its freezing temperature with \( H = H_2 \). When \( H_1 < H < H_2 \), the metal is at its fusion temperature, and such a region is called the mushy region in which metal is partially solid and partially liquid. The enthalpy form of the energy equation along with the temperature is shown below:

\[
\frac{\partial}{\partial r} \left( r \frac{\partial H}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{\partial H}{\partial \theta} \right) = \frac{1}{\alpha} \frac{\partial H}{\partial t}
\]

(5)

Note that these equations will be solved both in the solid and the liquid regions.

Numerical Analysis

An analytical solution of a heat transfer problem exists only for very simplified cases. This is due to the complexity of the heat transfer problem during die casting operation that involves partial solidification of the molten metal and formation of the moving solid-liquid interface. In order to derive the exact solution, a number of simplification assumptions were taken.

The idea of solving differential equations by a numerical method is quite old. It existed before the invention of the computer. Due to the complexity of calculations involved, it was limited to simple problems. With widespread use of computer technology, the numerical method became the prime method of solving complex physical problems.

The finite difference algorithm is based on replacing partial differential equations with algebraic equations. Explicit numerical approximation provides straightforward solutions of differential equations. It requires minimum computational effort. A disadvantage of this approximation is that it limits the size of the time step. The implicit method requires solution of the system of the algebraic equation, but it is stable at any time step size.

Hopscotch implicit–explicit numerical approximation is a combination of implicit and explicit replacement. It doesn't require solution of the system of algebraic equations and is unconditionally stable.

The first step of the finite difference method is to overlay the finite difference grid over the physical domain. In order to achieve a closer approximation to a physical domain, two numerical algorithms were developed. Since the shot cylinder was modeled as a long tube, with the effects of heat conduction from the ends being ignored, heat transfer problems in the shot sleeve were solved in a polar coordinate system. The die cast die cavity was approximated in a Cartesian coordinate system with heat transfer considered only in the direction perpendicular to the walls of the cavity.

Numerical Analysis of a Heat Transfer in a Shot Sleeve

In order to better match the geometry of the shot sleeve, calculations were conducted in a polar coordinate system. Physical domain is shown in Figure 2.

Figure 2 – Cross-section of a shot cylinder.
We will divide physical domain into an arbitrary grid. Distance between grid points are called steps. Each intersection point of a grid lines is called a node in which \( r = i \Delta r, \theta = j \Delta \theta \) and \( t = k \Delta t \). \( i \) and \( j \) are node numbers in \( r \) and \( \theta \) direction respectively, and \( k \) is time step number. Computational domain and boundary conditions are shown in Figure 3. The temperature and enthalpy denoted \( H \) and \( T \) respectively. Odd-even Hopscotch approximations at even points are:

\[
H_{i,j}^{n+1} = H_{i,j}^n + \alpha \left[ \frac{1}{\Delta r} \left( \frac{1}{r} \frac{\partial H_{i,j}}{\partial r} - 2 \frac{H_{i+1,j}^n + H_{i-1,j}^n}{r} \right) \right]
\]

At odd nodes:

\[
H_{i,j}^{n+1} = H_{i,j}^n + \alpha \left[ \frac{1}{\Delta r} \left( \frac{1}{r} \frac{\partial H_{i,j}}{\partial r} - 2 \frac{H_{i+1,j}^n + H_{i-1,j}^n}{r} \right) \right]
\]

And

\[
T_{i,j}^{n+1} = (H_{i,j}^{n+1} - L)/C
\]

In order to avoid singularity at the point \( r \to 0 \) as it was suggested in Ref.\(^1\), the coordinate system is transformed as it shown in Figure 4. In our case:

\[
r = \frac{(2i + 1)\Delta r}{2}
\]

This will allow one to avoid having a node at \( r=0 \) point.

In order to verify numerical code, the next case was studied, and results were compared to the results obtained by the general CFD code Flow3D.

Properties of general purpose die cast aluminum A380 were used for the analyses. The shot sleeve had a 0.12m diameter initially filled by molten aluminum at 660°C to 40% of its volume (Figure 1). In both cases, computational mesh was evenly spaced in both radial and azimuthal directions. Solutions were considered converged when the differences in values between consecutive runs were less than 0.1%.

Results of temperature distribution in the cross-sectional area of the shot cylinder are shown in Figure 5. Results of verification analyses between 2-D numerical algorithm and Flow3D are shown in Figure 6. Since error doesn't exceed 4%, the developed numerical code for the shot sleeve will be used in the article for further analyses. In order to verify numerical algorithm, mesh sensitivity analyses were conducted. Results of the analyses are presented in Table 1.

### Table 1 – Results of mesh sensitivity analysis.

<table>
<thead>
<tr>
<th>Number of nodes</th>
<th>200</th>
<th>150</th>
<th>90</th>
<th>70</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent of results variation</td>
<td>0.01</td>
<td>1.2</td>
<td>2.5</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

**Numerical Analysis of a Heat Transfer in a Cavity of a Die Cast Die**

Odd-even Hopscotch approximation at even points is:
\[ H^{i+1}_{j} = H^{i}_{j} + \alpha \frac{\Delta t}{\Delta x^2} (H^{i+1}_{i+1,j} - 2H^{i+1}_{i,j} + H^{i+1}_{i-1,j}) \quad (11) \]

At odd nodes:

\[ H^{i+1}_{j} = H^{i}_{j} + \alpha \frac{\Delta t}{\Delta x^2} (H^{i+1}_{i+1,j} - 2H^{i}_{i,j} + H^{i+1}_{i-1,j}) \quad (12) \]

Results of numerical calculation were verified against solution obtained in Flow3D. Liquid metal was considered stationary in 1-D algorithm, with constant temperature specified at an upper and lower boundary. For Flow3D analyses, liquid metal was moving at 89.6 m/s velocity. Temperature was considered constant on liquid metal —

die steel interface. Metal was at 660°C at t=0. Die steel temperature was 230°C. Results shown in Figure 7 are in good correlation between 1-D algorithm and Flow3D.

**Figure 7** — Result verification analyses in cavity of the die cast die.

**Thermal Analysis on the Slow-Shot Stage**

In order to examine ways to minimize temperature loss at the first stage of the die casting process, several parameters were investigated.

1. Percentage of fill of the cold chamber
2. Slow-shot velocity profile

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**Slow-Shot Velocity Profile**

Slow-shot velocity is limited by the maximum critical velocity. If plunger velocity exceeds critical, metal reaching the top of the cold chamber will then fall. This will cause air entrapment. Entrapped air will be pushed into the cavity.

In order to optimize the slow-shot stage of the die-casting process, two slow-shot profiles have been considered. Analytically calculated slow-shot velocity was optimized using commercially available code Flow3D®. Both constant and variable velocity profiles of the slow shot utilize the theory of flow in the open channels. Calculations were conducted with cylinder diameter at 120 mm, percentage of fill 40% and the slow-shot stroke at 1,000 mm.

Constant velocity profile:

\[ \frac{dX}{dt} = 2\left( \sqrt{gD} - \sqrt{g \eta} \right) \quad (13) \]

Variable velocity profile:

\[
X^* = \frac{c_0 + \frac{1}{2} X(t)}{c_0 + \frac{3}{2} X(t)} \tan(\alpha_{\max})
\]

where, \( c_0 \) is the velocity of the shallow wave traveling along the free surface due to gravity, \( X \) is plunger velocity, \( \alpha \) is a slope of the metal's free surface, \( L \) total stroke of the plunger, and \( g \) is acceleration due to gravity.

Velocity can be calculated by a numerical integration. Equations in this section are given in a final form. For more information, refer to the original references for detailed information.

In order to determine more preferable slow-shot profile, velocity was calculated using the same initial condition using both Equations 13 and 14. Initial calculation of the plunger velocity using Equation 8 shows that plunger critical velocity with the above-mentioned parameters needs to be 769 mm/s. Flow3D was used to verify and adjust flow velocity until proper fill of the cylinder reached. Final velocity, based on 3-D analyses, was calculated to be 381 mm/s. Time required for plunger to reach a position of a full cylinder was calculated in two stages:

- **Stage 1** — plunger velocity 127 mm/s for a first 127 mm
- **Stage 2** — plunger velocity 381 mm/s until plunger is full with metal (271 mm)

**Table 2** — Results of the temperature differences between two velocity profiles.

<table>
<thead>
<tr>
<th>Velocity Profile</th>
<th>Constant Velocity Profile</th>
<th>Variable Velocity Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity (mm/s)</td>
<td>381</td>
<td>788 (final)</td>
</tr>
<tr>
<td>Time (s)</td>
<td>1.71</td>
<td>1.46</td>
</tr>
<tr>
<td>Temperature °C</td>
<td>586.9</td>
<td>595.4</td>
</tr>
</tbody>
</table>

**Percentage of Fill of the Cold Chamber of a Die Cast Machine**

**Table 3** — Influence of percentage of fill on final metal temperature.

<table>
<thead>
<tr>
<th>Percentage of Fill</th>
<th>Time to Plunger Full Position</th>
<th>Final Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1.66</td>
<td>588</td>
</tr>
<tr>
<td>40</td>
<td>1.46</td>
<td>595.4</td>
</tr>
<tr>
<td>50</td>
<td>1.39</td>
<td>597.8</td>
</tr>
<tr>
<td>60</td>
<td>1.21</td>
<td>604</td>
</tr>
<tr>
<td>70</td>
<td>1.01</td>
<td>611</td>
</tr>
</tbody>
</table>

**Fast-Shot Analyses**

The numerical algorithm, presented in this paper, used to calculate metal temperature during fast-shot stage of die cast process was verified using Flow3D and the exact solution of Stefan's problem of solidification. See Appendix A for the mathematical definition of the exact solution

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Conclusions

The one-dimensional algorithm presented in this paper can be incorporated into the die cast process design in conjunction with 3-D CFD analyses on fast and slow stages of the die casting process. It will allow one to substantially reduce the number of iterations required to achieve desired process parameters. Slow shot with acceleration proposed allows one to achieve desired flow in a shot sleeve at a shorter time compared to constant velocity profile. Percentage of fill in a shot sleeve has to be optimized to reduce temperature losses on the first stage of the die casting process. The higher percentage of fill corresponds to a shorter slow-shot stage and as a consequence to a less loss of heat. The impact of excessive heat loss to the metal in the sleeve is significant to casting quality. It generates higher percentages of solids in the casting in erratic and unplanned formations. These can be found in the form of cold flakes, lamination, within the casting sections and breakout, segregations and cold flow on the casing surfaces. These conditions reduce properties, compromise integrity and may render the casting unusable. Managing the heat loss in the sleeve reduces the occurrence of these problem conditions.


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References


Appendix A

To view Appendix A visit www.diecastingengineer.org/issues/files/reikher.pdf.

About the Authors

Alexandre Reikher is a product design leader at Albany Chicago Co. in Pleasant Prairie, WI, the company he joined in 1997. He is responsible for all aspects of product design, finite element and computational fluid dynamics analysis. He has 25 years experience in die casting industry. He holds five patents and co-authored the book Casting: An Analytical Approach.

Harold Gerber (Hal) is the sales, engineering and technology leader at Albany Chicago Company LLC, a mid-sized pressure die casting and machining operation in Pleasant Prairie, WI. He has worked in the die casting industry since 1974, developing designed applications for machined pressure die castings. He earned a BS in organizational behavior (with math, science minors) and an MS in engineering management, both from Northwestern University. He is a long-standing member of NADCA, its predecessor SDCE and currently serves on the NADCA Board of Governors. Gerber has been a participant in the Process Development and the Computer Modeling Task forces and shares a patent with ACC on casting insert technology.

Dr. Krishna M. Pillai is associates professor at University of Wisconsin – Milwaukee. He is also the director of Laboratory for Flow and Transport Studies in Porous Media at UW. His research interests lie in several areas of porous media transport, including flow and transport in fibrous media, working in rigid and swelling porous media and evaporation modeling using network and continuum models. He was awarded the prestigious CAREER grant in 2004 by the National Science Foundation of USA to model and simulate flow processes during mold filling in liquid molding processes used for manufacturing polymer composites.

Tien-Chien Jen is professor and chairman of the Mechanical Engineering Department at the University of Wisconsin – Milwaukee. He earned a Ph.D. in mechanical engineering from the University of California – Los Angeles in 1993 and has worked at the University of Wisconsin since 2001. He is the author of numerous publications, and his research focuses on fuel cells and hydrogen generation technology, biofuel and biodigester design process optimization, solar energy harvesting, thermal aspects of machining processes, cold gas dynamic spraying surface coating technology, environmentally benign machining processes, diffusion bonding, nano-technology (thermal) in materials processing and biosensing technology.