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The Sponge Layer Method in *FLOW-3D*

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I. Theory and Method

In coastal and ocean engineering applications, a computational domain with a limited size and open boundaries is often used to simulate periodic wave propagation in open water. When a wave train moving through the domain reaches an open boundary, a proper boundary condition must be used to minimize wave reflection. Otherwise, incorrect wave shape, severe water volume change and unphysical wave-structure interaction may occur. In general, two types of methods exist to reduce wave reflection at open boundaries: the radiation boundary condition and the sponge, or wave-absorbing, layer method.

The radiation boundary condition was originally proposed by Sommerfeld (1912) for mathematical physics and later revised by Orlanski (1976) for hydraulic flow. It was implemented in ***FLOW-3D*** as “the outflow boundary condition” (Hirt, 1999) and has been used for many successful applications since then. The idea is to allow continuous wave propagation through an open boundary. The commonly used equation is

$$\frac{\partial q}{\partial t} + C\frac{\partial q}{\partial x} = 0 \qquad (1)$$

where q is a dependent physical quantity, $+x$ points out of the boundary, and C is the local phase speed.

The radiation boundary condition has its limitations, however. Equation (1) is obtained from the assumption of a linear wave, and C is numerically evaluated at the open boundary. Theoretically, this method is not suitable for nonlinear wave and dispersive wave conditions. Although short-term wave reflection from an open boundary can be reduced to a small amount by the radiation boundary condition, an accumulation of the wave reflections over long periods can become significant and affect the wave motion inside the computational domain. Figure 1 shows wave profiles at different times for a 2D nonlinear wave with waves generated at the left side and an outflow boundary placed at the right side of the domain. The wavelength is 10.43 m, wave period 2.8 s, wave height 0.4 m, and the undisturbed water depth is 2 m. The wave takes 13.42 s to travel from the one side to the other side of the domain. It can be seen that the calculated wave profile becomes more irregular with time. Figure 2 shows that the calculated water volume in the domain increases significantly with time. At 75 s, the computational domain is completely filled with fluid.

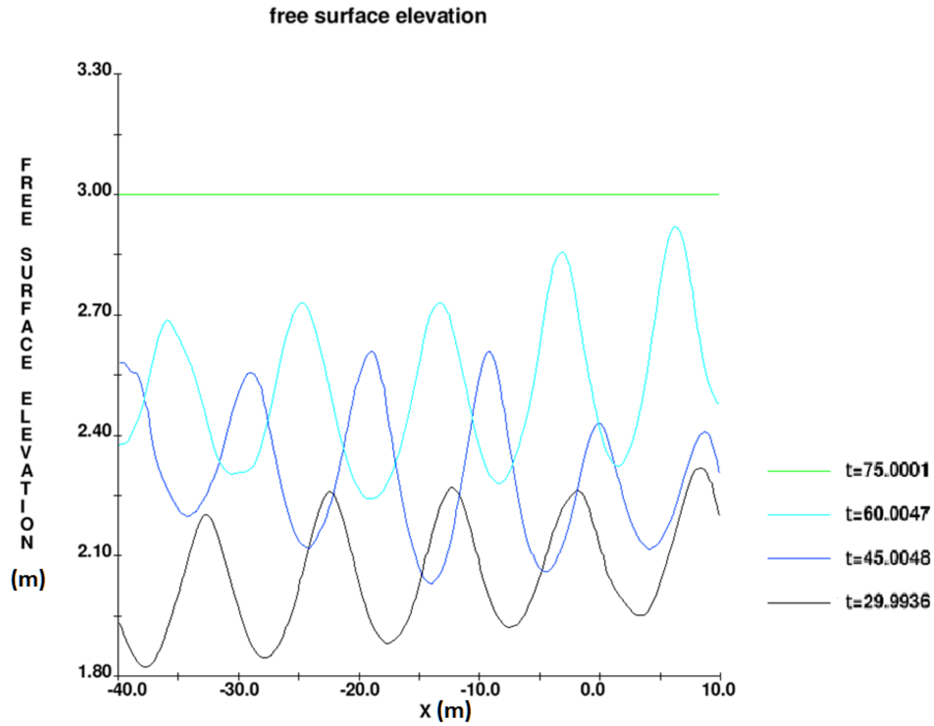


Figure 1. The wave profile versus time using the radiation boundary condition.

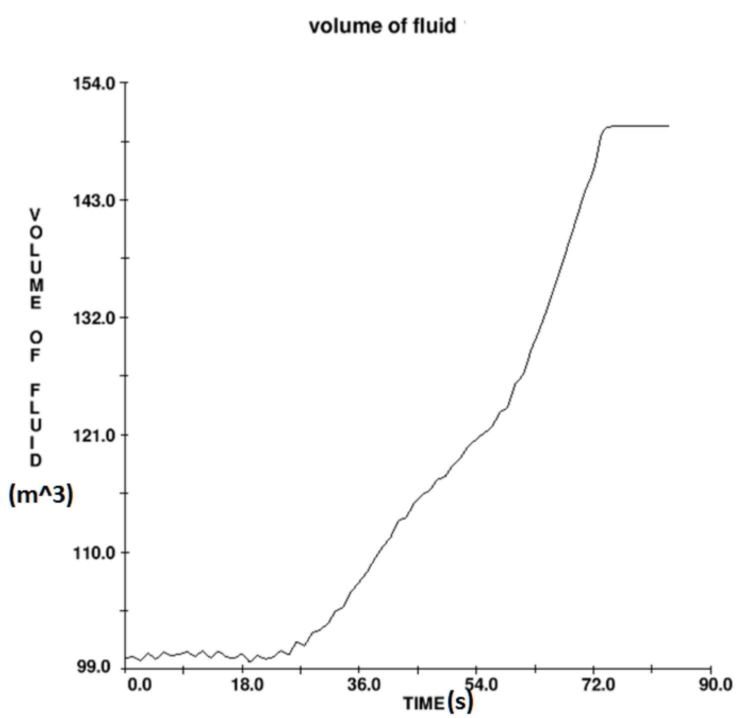


Figure 2. The total water volume versus time using the radiation boundary condition.

Unlike the radiation boundary condition, the sponge layer method uses a wave-damping region called a *sponge*, or *wave-absorbing*, layer to absorb waves before they reach an open boundary. The region starts somewhere inside the computational domain, ends at the open boundary and extends fully in the vertical direction, as shown in Figure 3. Inside the sponge layer, a wave is dissipated by an artificial damping force, reducing the potential for its reflection at the boundary. The damping force is typically linear with respect to fluid velocity. The damping coefficient is empirical and can be location dependent (Davies, 1976, 1983). A sponge layer is most effective for reducing wave reflections at open boundaries if it is used in combination with the radiation boundary condition (Israeli, 1981).

In this work, a sponge layer approach is implemented in ***FLOW-3D*** v11.1. In the sponge layer, the Navier-Stokes equation is modified as

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho} \nabla p + \nabla \cdot \nabla(\nu \vec{u}) - k(\vec{u} - \vec{u}_{str}) \quad (2)$$

where $-k(\vec{u} - \vec{u}_{str})$ is the artificial damping force that dissipates the wave motion, k is the damping coefficient in units of $(\text{time})^{-1}$, and $\vec{u}_{str}$ is the background stream velocity that is exempted from damping.

The coefficient k can be constant inside the sponge layer or increase linearly in the wave propagation direction (***n*** in Figure 3, constant for a given sponge layer). It is evaluated using

$$k = k_0 + l \cdot \frac{k_1 - k_0}{d} \quad (3)$$

where k_0 and k_1 ($k_1 \geq k_0$) are the values of k at the starting side of the sponge layer and the open boundary, respectively. The distance l is measured from the starting side of the wave-absorbing layer toward the open boundary (in the direction of ***n*** in Figure 3). Finally, d is the length of the sponge layer.

A sponge layer is defined by a special geometry component called the *wave-absorbing component* in ***FLOW-3D*** v11.1. It is completely open to fluid flow but applies damping to wave motion. The component should extend to the open boundary through which the wave moves out. For flexibility, the component can be defined with multiple subcomponents using both STL files and primitives and its shape can be arbitrary. In most applications, however, only a simple rectangular block is needed. Numerically it is treated in the solver in the same way as flow losses in porous media.

The required input parameters are the location of a point P on the starting side of the sponge layer, ***n***, k_0 and k_1. Velocity $\vec{u}_{str}$ is an optional input parameter. It should be defined if its value is known, *e.g.*, $\vec{u}_{str} = 0$ if no mean stream exists; otherwise, it is calculated at each time step as the average fluid velocity inside the sponge layer over the previous wave cycle. During the first wave cycle, $\vec{u}_{str} = 0$ is assumed. It is noted if the value of $\vec{u}_{str}$ is known (either zero or non-zero), users should specify that value because a calculated $\vec{u}_{str}$ may have more or less deviation from its known value due to computational errors.

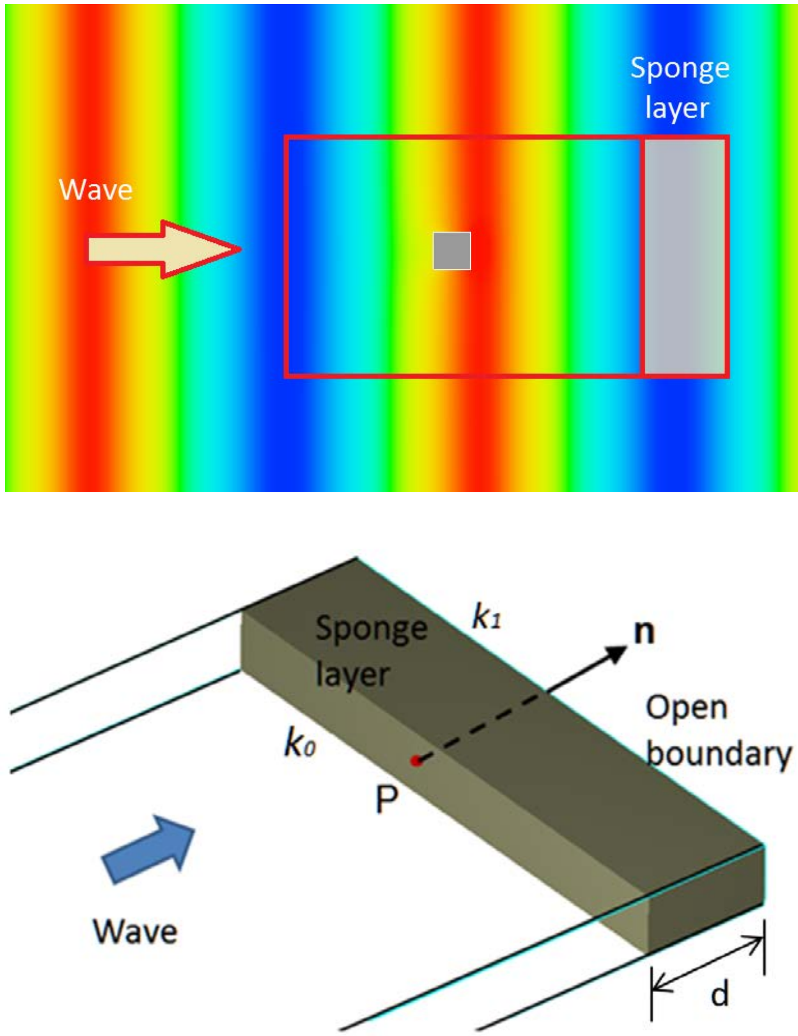


Figure 3. A sponge layer and the artificial damping coefficient.

II. Method Verification

1) 2D Test

The sponge layer approach is tested for a 2D nonlinear wave simulation shown in Figure 4. The wavelength is 10.43 m, wave period 2.8 s, wave height 0.4 m, and the undisturbed water depth is 2 m. The background stream velocity is zero. Initially water is stationary with surface elevation at z = 2.0 m. The wave train enters a 50 m long computational domain in the +x direction from its left boundary at $x = -40$ m. The right boundary, at $x = 10$ m, is open and the radiation boundary condition is used there. A sponge layer is placed before the open boundary from $x = 0$ to 10 m. Inside the sponge layer, the damping coefficient increases linearly in the +x direction from 0 at $x = 0$ to 1.0 s^{-1} at $x = 10$ m.

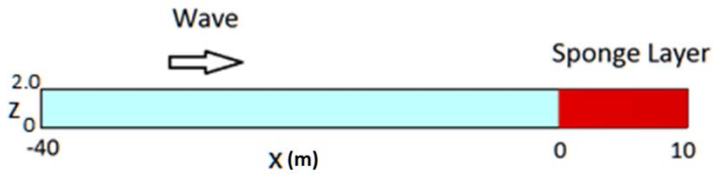
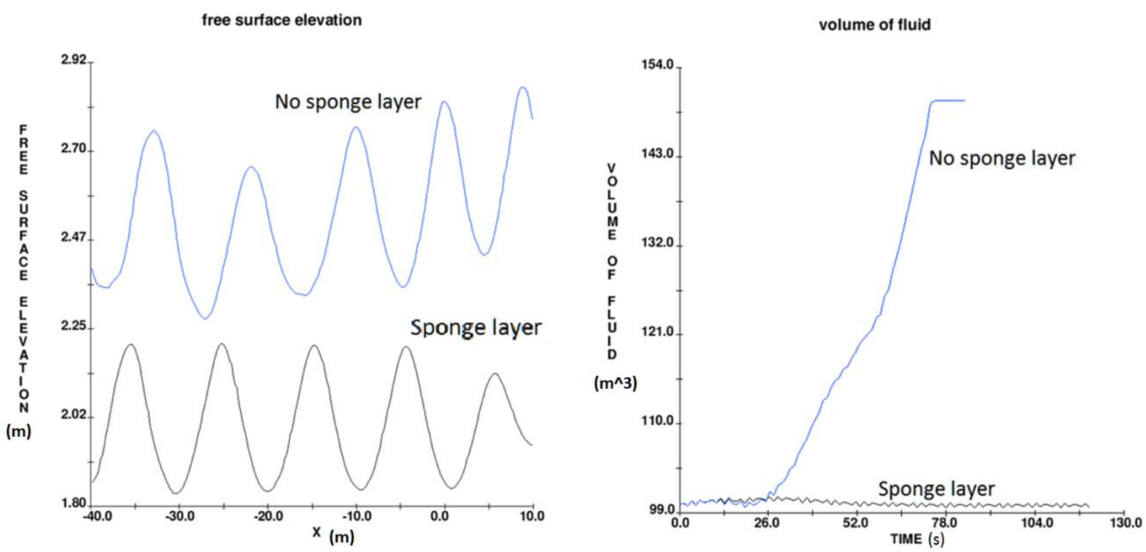


Figure 4. Sponge layer setup in a 2D computational domain.
A nonlinear wave moves in the +x direction.

In Figure 5, the simulation result is compared with the result obtained without sponge layer. It can be seen that in the case with the sponge layer the wave profile stays regular before the sponge layer. The wave decays inside the sponge layer between x=0 and 10 m due to the artificial damping. In the case without the sponge layer, with time the wave profile changes to an irregular shape. It is also found that water volume conservation is significantly improved when the sponge layer is used.



(a) (b)

Figure 5. Comparison of simulation results with and without a sponge layer.
(a) Wave profile along x at t=60 s. (b) Water volume variation with time.

2) 3D Test

A 3D test is conducted for the interaction of a nonlinear wave with a moored platform in open sea. The platform deck size is 78 m × 77 m. The total height of the platform is 105 m. The incoming wave in the +x direction has a 10 m wave height, 8 s period and 100 m wavelength. The water depth is 500 m. The computational domain is 420 m long (in *x*), 200 m wide (in *y*) and 130 high (in *z*), as shown in Figure 6. To save computational effort, the initial depth of water is 50 m in the computational domain while a free-slip condition is used at the domain's bottom boundary. This is a reasonable approximation because wave motion is typically negligible half wavelength below water surface. A 100 m long sponge layer is placed immediately before the downstream open boundary where the radiation boundary condition is used. The damping coefficient increases linearly from 0 to 1.0 s^{-1} in the sponge layer in the wave propagation direction. Numerical simulations are conducted for activated and deactivated sponge layer, respectively.

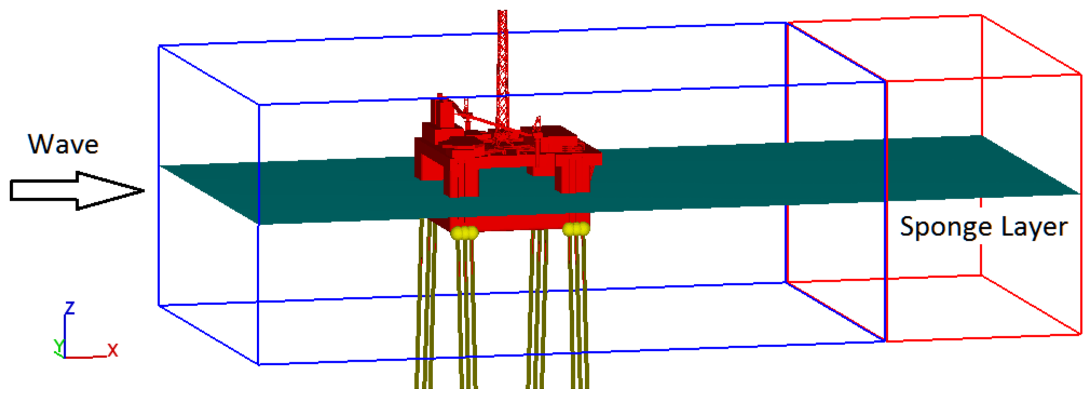
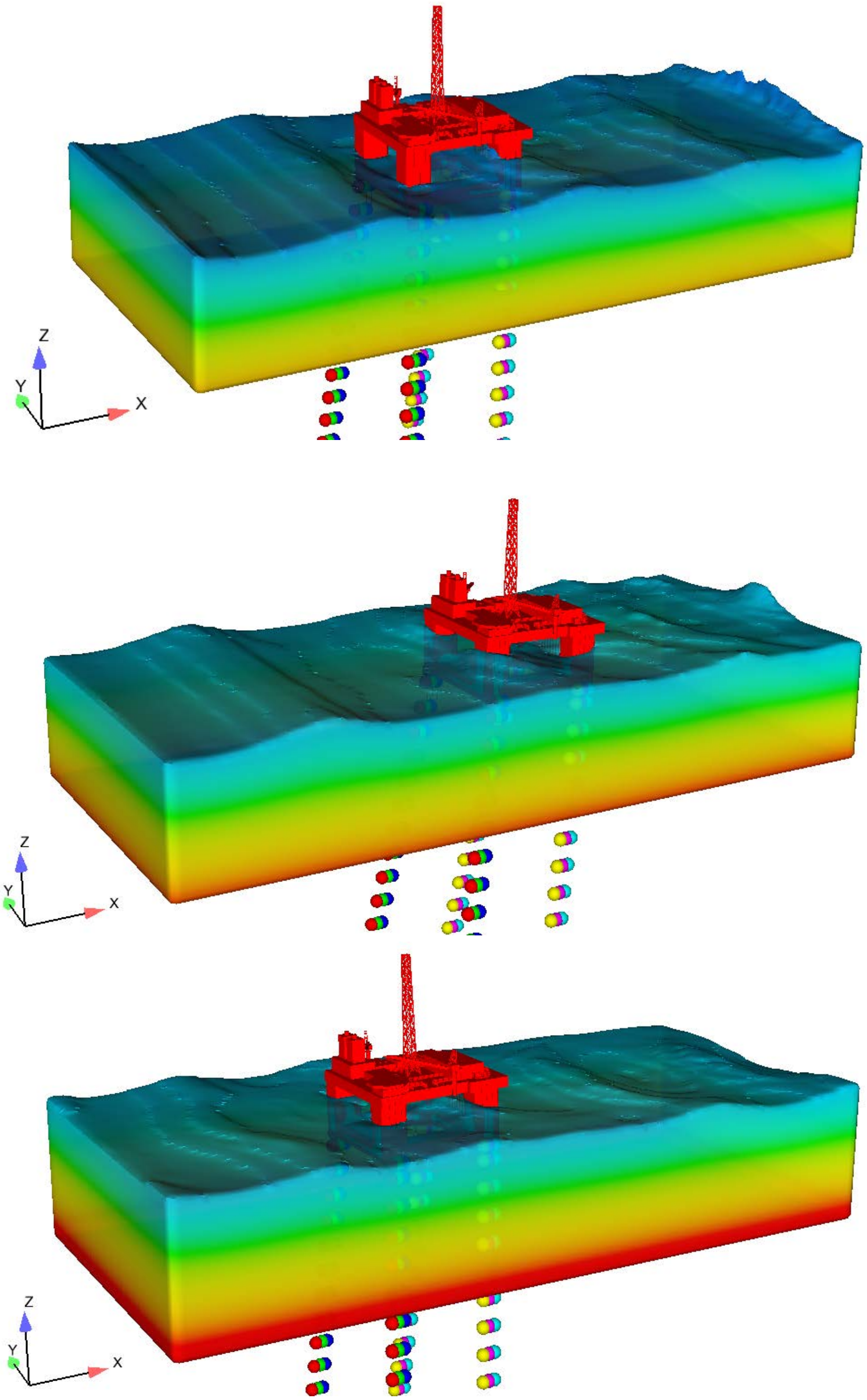


Figure 6. Computational domain and sponge layer

Figure 7 shows the calculated wave and the platform motion at 60 s, 120 s and 300 s when the sponge layer is deactivated. Obvious wave reflection can be seen at the open downstream boundary. From 120 s to 300 s, the platform moves back against the wave propagation direction, which is not realistic. Figure 8 shows the corresponding result when the sponge layer is activated. It can be seen that wave shape is maintained in the computational domain except in the sponge layer where the wave is artificially damped. The platform does not experience an unphysical backward movement, and no obvious water volume increase is observed.

Figure 9 presents the variation of water volume with time. When sponge layer is turned off, the water volume constantly increases with time, reaching 16.6% above the initial value at 600 s. With the sponge layer is turned on, water volume is stable, increasing only 0.98% by 600 s.

Figure 10 shows the time variation of the x-coordinate of the platform's mass center. When the sponger layer is turned off, the platform drifts downstream for about 120 s, but then it unphysically moves back toward its initial location. This indicates a strong wave reflection from the open boundary after certain time in the simulation, even although the radiation boundary condition is used. When the sponge layer is turned on, the platform is drifts for 120 s, and then its location is stabilized with mild fluctuations.



(60 s)

(120 s)

(600 s)

Figure 7. A wave passing around a moored platform, when a radiation boundary condition is used but not the sponge layer. Color denotes pressure.

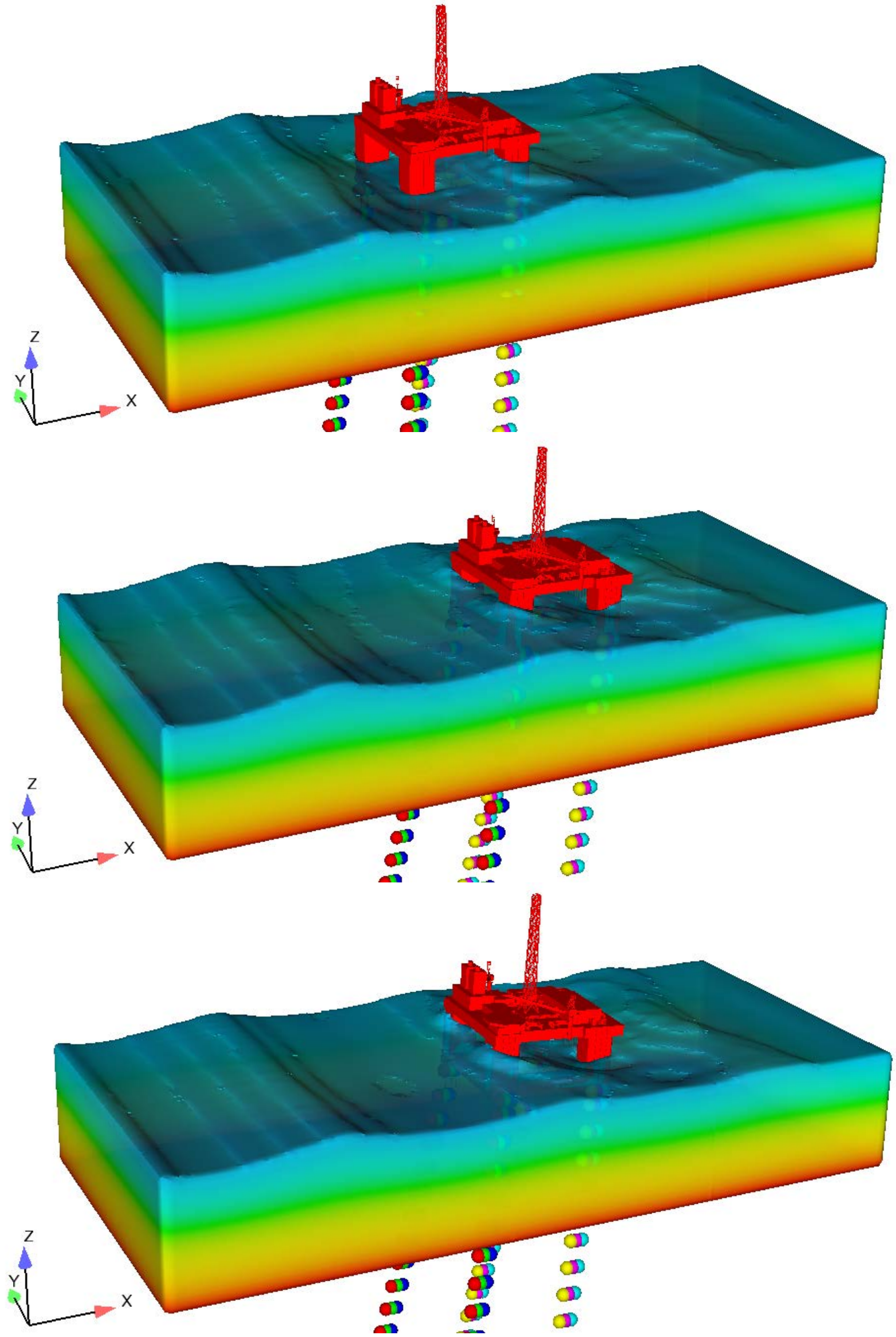


Figure 8. A wave passing around a moored platform, with both a radiation boundary condition and a sponge layer present. Color denotes pressure.

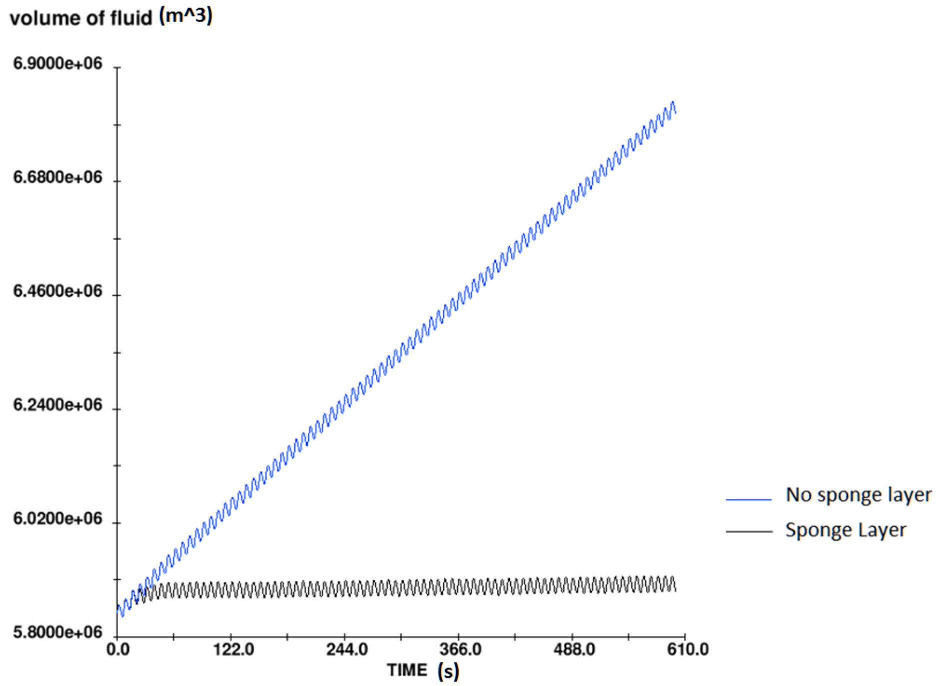


Figure 9. Time variation of the water volume for simulations with and without a sponge layer.

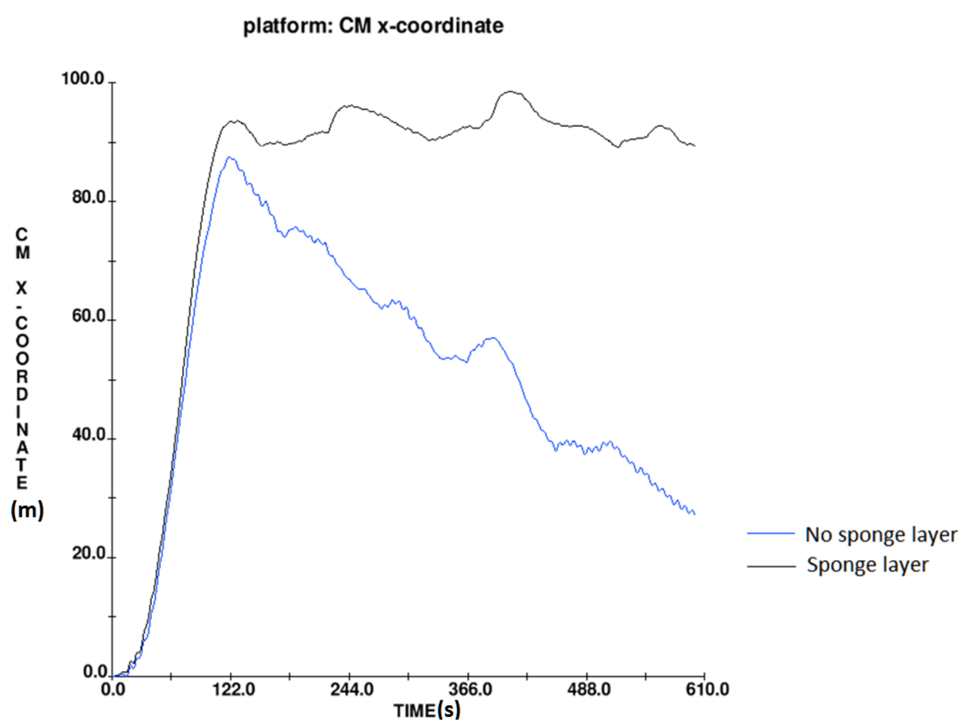


Figure 9. Time variation of the x-coordinate of the platform's mass center.

III. Parametric Study

A parameter study has been conducted to investigate the effectiveness of the sponge layer for wave reflection at an open boundary. The computational domain used for the simulations is shown in Figure 4. A uniform mesh in horizontal direction is used. The wave parameters are the same as those in the 2D test in the preceding section.

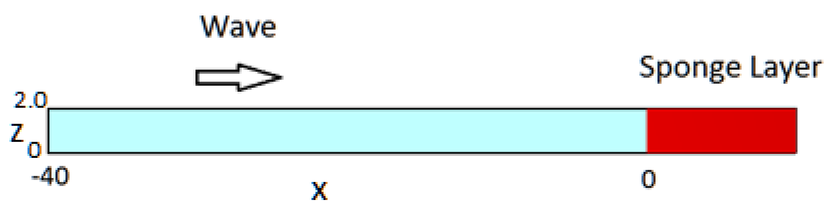


Figure 10. 2D computational domain and sponge layer

1) <u>Sponge Layer Length</u>

Simulations are conducted with different length of the sponge layer. Mesh is uniform in the x direction. The radiation boundary condition is used at the open boundary. The damping coefficient increases linearly in the $+x$ direction from 0 to 1.0 s^{-1} in the sponge layer. Three tests are carried out with the sponge layer length specified as 2, 5 and 10 m, respectively. Figure 11 shows the calculated wave profiles at 120 s and time variation of water volume.

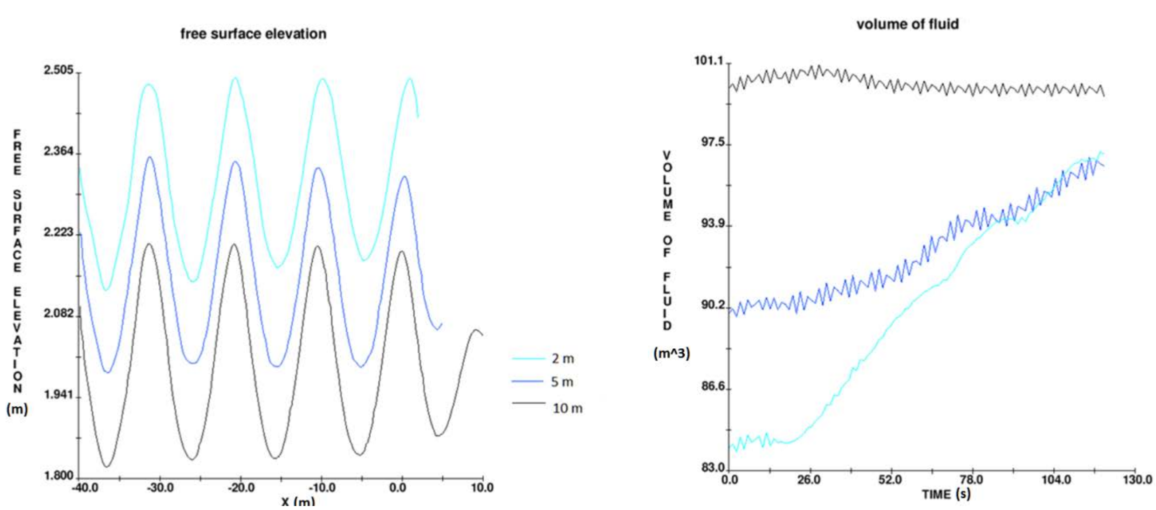


Figure 11. Wave profiles at 120s and variation of water volume with time for different sponge layer lengths.

A longer sponge layer performs more efficiently to damp waves and gives better wave shape and mass conservation. On the other hand, a longer sponge layer length takes more computational effort. It is found that the sponge length equal to about one wavelength yields good wave profile and the best volume conservation in the test. Note that the different initial water volume for different sponge layer length is due to the different sizes of the sponge layer.

2) Damping Coefficient

Simulations were conducted with different damping coefficient distributions in the sponge layer: (1) uniform k=0.5 s^{-1}, (2) uniform k=1.0 s^{-1}, and (3) a linear increase from k_0=0, to k_1=1.0 s^{-1}. The same computational domain was used as in the previous test, with the sponge layer length of 10 m, about one wavelength of the nonlinear wave. The calculated wave profiles at 120 s and the variation of the water volume with time are shown in Figure 12. It can be seen that the linear increase of the damping coefficient from 0 to 1 s^{-1} gives the best volume conservation and wave profile.

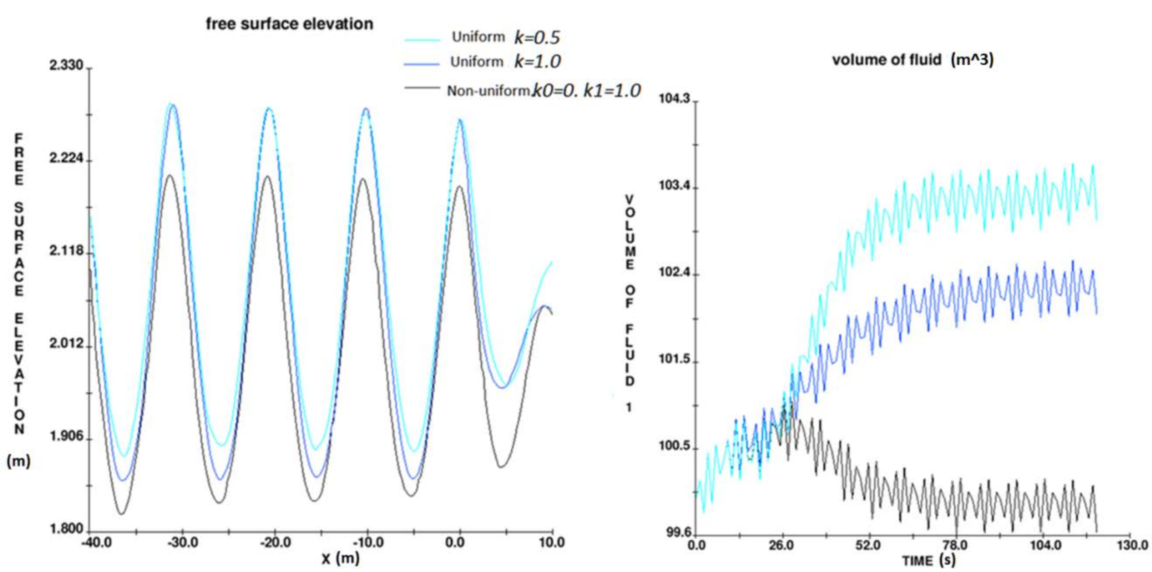


Figure 12. The wave shape at 120 s and time variation of the water volume for different damping coefficient distributions.

3) Mesh in Sponge Layer

Simulations were conducted with different mesh size distribution in the horizontal direction inside the sponge layer: (1) the same uniform size as that before the sponge layer, (2) mesh size increases linearly and is doubled at the open boundary, and (3) mesh size increases linearly and is tripled at the open boundary. The sponge layer length is 10 m, about one wavelength of the nonlinear wave. The damping coefficient increases linearly in the wave propagation direction from 0 to 1.0 s^{-1} in the sponge layer. The calculated wave profiles at 120 s and variation of the water volume with time are shown in Figure 13. It can be seen that the uniform mesh gives the best volume conservation and wave profile. The non-uniform mesh with tripled size at the open boundary gives the worst result.

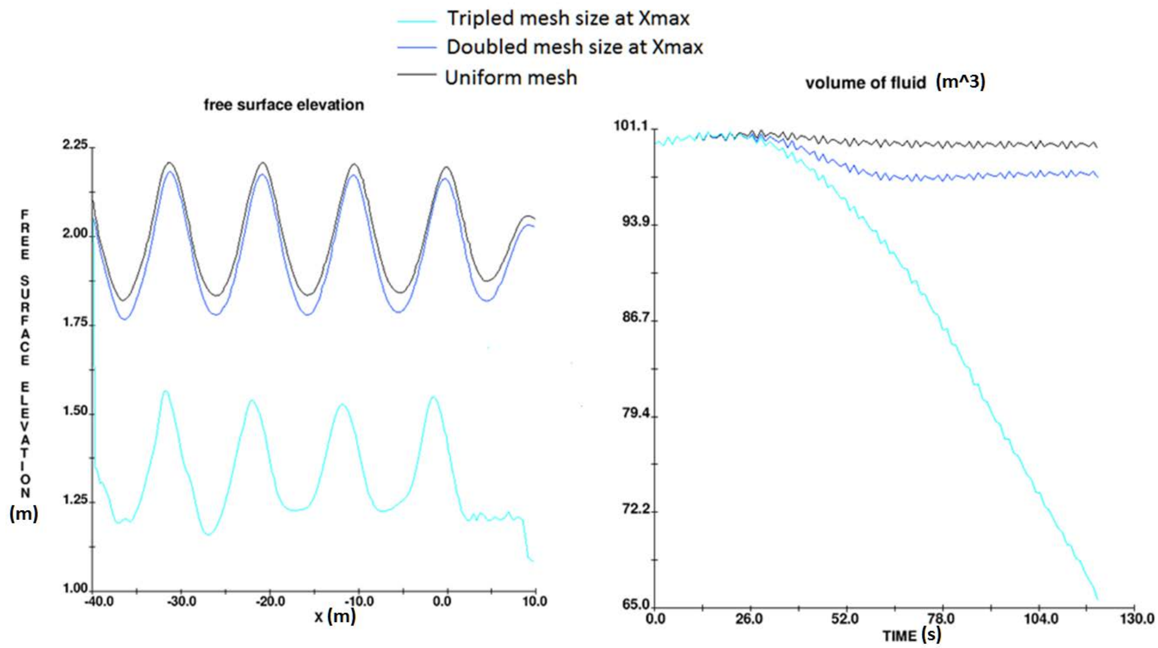


Figure 13. Wave profiles at 120 s and time variation of water volume for different mesh size distributions in the horizontal direction in the sponge layer

4) Boundary Condition at Open Boundary

Simulations were conducted for two different boundary conditions at the open boundary: (1) a radiation boundary condition, and (2) a continuative boundary condition (zero-gradient of physical quantities). The sponge layer length is 10 m, about one wavelength of the nonlinear wave. The damping coefficient increases linearly in the wave propagation direction from 0 to 1.0 s^{-1} in the sponge layer. Uniform mesh size in the horizontal direction is used throughout the domain. Figure 14 shows the calculated wave profiles at 120 s and time variation of the water volume. The radiation boundary condition combined with sponge layer gives much better wave profile and water volume conservation. This is consistent with Israeli (1981) that a sponge layer is most effective to reduce wave reflection at an open boundary if it is combined with the radiation boundary condition.

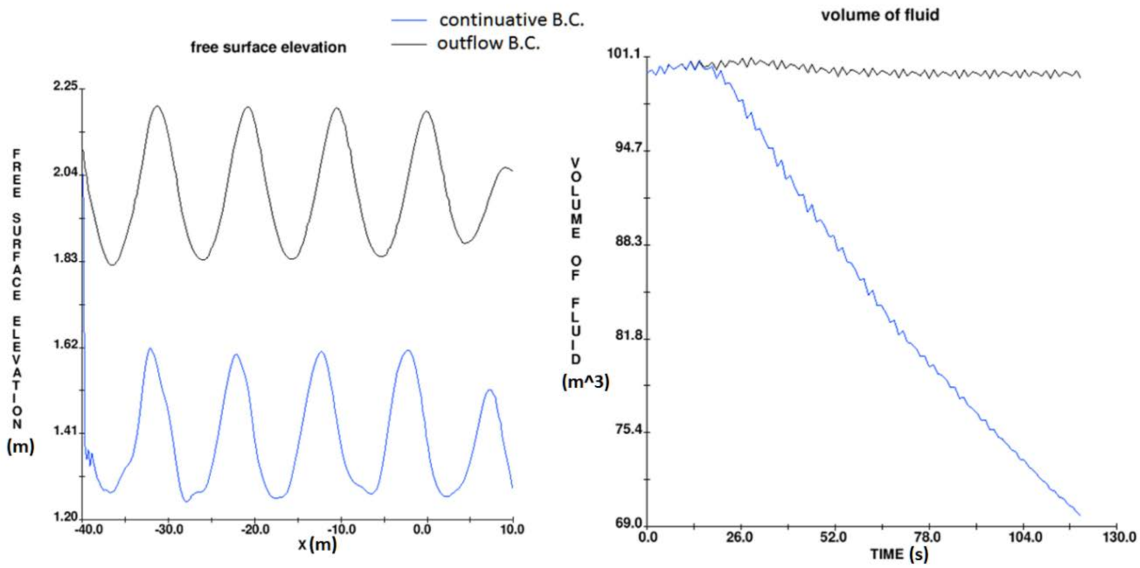


Figure 14. Wave profiles at 120 s and time variation of the water volume for different boundary conditions at the open boundary.

IV. Conclusions

It has been found that the use of the radiation boundary condition (also called *outflow* boundary condition in ***FLOW-3D***) fails for long-time simulations of periodic surface wave propagation in open water, resulting in irregular wave shapes, water mass conservation issues and incorrect wave-structure interactions. In this work, a sponge layer method is investigated and implemented in ***FLOW-3D*** version 11.1**.** A sponge layer is placed immediately before an open boundary, applying linear artificial damping to dissipate wave motion before it reaches the boundary. It is found that a combination of the sponge layer and the radiation boundary condition is most effective to minimize wave reflection at the boundary, maintains accurate wave shape and conserves water volume in the computational domain. Other conclusions are:

1) Uniform mesh size inside a sponge layer in the wave propagation direction works better than gradually stretched mesh size and is thus recommended for a sponge layer.
2) A longer sponge layer gives better results but consumes larger computational effort than a shorter sponge layer due to more mesh cells that are used. It is recommended the sponge layer length be at least one wavelength of the simulated wave.
3) A linear increase of the damping coefficient in the wave propagation direction is recommended in the sponge layer. It is found that the values of 0 and 1.0 s^{-1} for the minimum and the maximum values of the damping coefficient give good results.

The above recommendations for sponge layer parameters are found empirically using relatively simple calculations. Further parameter calibrations may be needed to optimize the sponge layer performance in more complex situations, for example when multiple outflow boundaries and sponge layers are present, or when a random wave generator is used.

References

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