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A *FLOW-3D*® Continuum Model for Granular Media

01/14/13 C.W. Hirt

Background

Many types of granular media are encountered in processing and manufacturing industries. Because of its unusual properties, granular material can often pose difficult problems for engineers seeking to transfer, mix or otherwise manipulate it for useful purposes.

We present here a continuum model for computationally predicting the dynamics of a concentrated granular material. In this model, the designation of concentrated granular flow means the volume fraction of the granular material is often 50% or greater. The model is referred to as a "continuum" model because it is based on a continuous fluid representation of the granular solid, making no attempt to treat individual particles. The model utilizes many existing capabilities of the ***FLOW-3D***® computer simulation program, which have been modified and enhanced for granular flow.

Overview of Continuum Model

We begin with a general description of the model. Some details of specific elements are contained in the following section, which is then followed by computational test results.

Granular flows come in two basic types involving a mixture of solid particles surrounded by either a gas or liquid. The solid/gas mixture is the most common and is the focus of the first part of this document. Solid/liquid mixtures are treated in the second part.

A mixture of solid particles and gas (typically the gas is air), which is itself surrounded by pure gas, is a two-phase flow in which the air and solid material flow with their individual velocities, but are coupled through momentum exchanges resulting from pressure and viscous stresses. In high solid concentrations a strong coupling exists between the solid particles and the air so the mixture can be approximately modeled as a single, composite fluid [1]. Two-phase effects resulting from differences in the velocities of the two materials are accounted for using an additional approximation for their relative velocity that is referred to as a Drift-Flux model.

This composite and relative velocity approach has been selected as the basis for the model described in this note. It is assumed that the solid/fluid mixture can be represented as a single fluid with a sharp free surface at boundaries (interfaces) with pure air. The composite fluid, however, is allowed to have a non-uniform density.

The density of a mixture fluid element cannot change with the mixture flow because of the assumption of incompressibility. However, density variations may develop because of the relative flow between fluid and solid within the mixture. This relative flow, also considered incompressible, is modeled using a Drift-Flux approximation that is based on pressure gradients and viscous stresses acting on individual solid grains (with an empirical adjustment made for multiple particle interactions). A detailed description of the Drift-Flux model used in the ***FLOW-3D***® program is given in reference [2].

With the Drift-Flux model solid packing is accounted for, but an additional mechanism is required in the case of solid/gas mixtures to remove the gas displaced as the solid is compacted. After all, the initial solid-gas mixture is treated as incompressible, so if solid is compacting, then the final volume of the mixture must be less than its initial volume. In the current model this loss of mixture volume is treated as a loss of air at the free surface of the mixture. Air is removed and the free surface adjusted by computing a drift-flux slightly below the surface and assuming the thus computed air flux to have escaped from the mixture surface. The removed air is transferred to the adjacent pure air region, which may be a confined bubble.

In the case of a solid/liquid mixture there may be an interface (free surface) between the liquid and air, whether or not the liquid contains solid material. This is an essential difference in the two types of granular flow characterized by either gas or liquid surrounding the solid particles.

Another element important in modeling two-phase flow of solid/fluid is to have a mechanism for solid particles to collide and exchange momentum in addition to exchanging momentum by viscous stresses developed in the surrounding fluid. For our continuum model we have used a combination of flow resistance based on observations of blowing sand and sand dunes by Bagnold [3] and use a viscosity for the solid/fluid mixture that is modeled by an empirical expression generalized from the work of Bagnold [4]. The mixture viscosity includes both fluid viscosity and particle collision effects.

Furthermore, as solid packs to a density where individual grains begin to touch one another, it becomes more difficult for the solid/fluid mixture to flow. This state is sometimes referred to as one of mechanical jamming. At still higher solid densities, corresponding to solid packing where sand grains remain in contact with their neighbors, the solid is not able to flow at all and it is difficult for solid particles and the surrounding fluid to exchange places, that is, to have a relative drift velocity. This near fully packed state is represented as though the mixture has solidified

In summary, the continuum granular flow model consists of a single fluid representation of the solid/fluid mixture. The mixture may have a sharp interface with the surrounding air and may have a variable density. Density variations are computed in terms of a two-phase flow that allows for solid packing or un-packing depending on the forces in the flow. And finally, a feature important for some applications, especially those having a liquid component, is the inclusion of a dispersive pressure (see Bagnold [6]) created by particle collisions.

Taken together these features of the granular flow model address the main elements expected for the dynamic behavior of a high-density solid/fluid mixture. In particular, the model includes localized packing, venting of air at a free surface, a viscosity that depends on density and shear stress as solid density is increased to its close packing limit. The model also includes a close-packed or jammed state in which no flow is possible, but which still allows for an avalanche type of behavior.

Illustrative Examples Part I: Solid/Gas Mixtures

It is instructive to consider some simple test cases for granular flows that highlight basic features in the continuum model. In particular, the following examples provide insight into the model's advantages as well as some limitations.

The solid density of the granular material is that of sand, 2.6 gm/cc. The viscosity of air is taken as 1.7e-4 g/cm/s, and its density is 0.001g/cc. The viscosity of the mixture is computed from an empirical expression found to apply to many granular flow situations. Finally, the diameter of the sand particles has been set at 0.025cm unless otherwise specified.

Hydrostatic Test

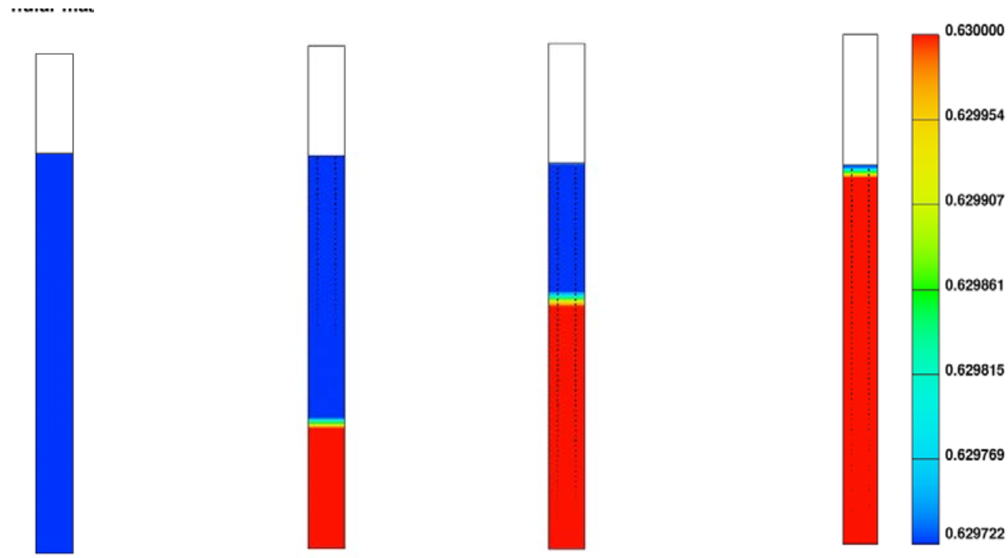
Mixed sand/air of initially uniform density 1.456 g/cc (loose packing solid volume fraction of 0.56) and height 10 cm is placed in a cylinder with a closed bottom. Normal gravity of 980.0 cm/s^2 should set up a hydrostatic pressure in the fluid and the resulting sand density should respond to the pressure gradient by packing at the bottom of the column and thinning at the top.

The settling of the sand in this case can exist in two modes. In addition to a solid fraction for close packing (0.63) the granular flow model also involves a solid fraction for jamming (0.6) and a solid fraction for random loose packing (0.55). Random loose packing occurs when the grains are touching just enough to support their weight under gravity provided there is no disturbance that drives them into a closer packed arrangement. In the present model a sufficient disturbance is the existence of a mixture velocity in excess of a few percent of the critical velocity at which flowing grains can dislodge grains in a packed bed [3]. Because the hydrostatic setup of a sand/air mixture in a vertical cylinder involves no mixture flow (i.e., an incompressible mixture in hydrostatic equilibrium) the sand will not settle to more than the random loose pack concentration. If the mixture is initialized with a packing fraction greater than the random loose pack value (0.55) there is no settling at all. If it is initialized to some value less than this value it will settle enough to reach random loose packing.

To have complete settling it is necessary to reduce the "Bagnold" critical velocity to zero, which is easily done using an input constant (*cvelgrn=0*). When this is done the sand will all accumulate at the bottom of the cylinder at the close packing density of 1.638g/cc. As the sand settles, the air is escaping at the free surface at the top thus reducing the height of the mixture in the cylinder. Figure 1 shows computed results at several times.

The settling (packing at the bottom) is clearly seen. After 0.5s these changes have only progressed into the column a short way. In the upper portion of the column there is no change in density because the drift-flux model preserves volume. In this region the sand

and air are uniformly slipping pass one another without changing the density. The maximum density of sand at the bottom of the column is about 1.638g/cc, which is a value close to the maximum packed value.



t=0.0s t=0.5s t=1.0s t=1.5s

Figure 1. Color is solid fraction of mixture: 0.55 initially and 0.63, the maximum close packing after settling. Gas escape at top can be seen to reduce the height of the column.

Heap Test

Everyone is familiar with sand piles that typically have a conical shape. For a given type of sand the angle of the side of the pile cone with respect to the horizontal is a constant referred to as the angle of repose. For the second test of the continuum model a simple two-dimensional, axisymmetric cylinder or heap of sand was simulated to see if the model would predict the expected slump of the sand into a conical pile having sides at a specified angle of repose.

The initial sand/air mixture was a cylinder 10.0cm in diameter and 10.0cm high resting on a horizontal surface and at the close packing volume fraction of 0.63, or density 1.638 g/cc. The angle of repose was specified to be 34°. Flow is induced by gravity alone and the simulation was run for 10.0s, requiring a CPU time of about 58s.

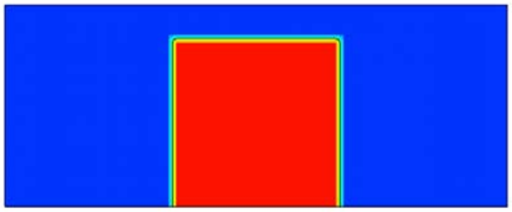


Figure 2a. Red is sand mixture, which is close to maximum packing. Time 0s.

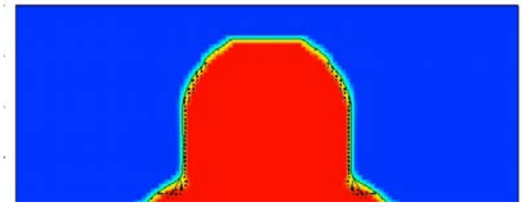


Figure 2b. Time 1.0s.

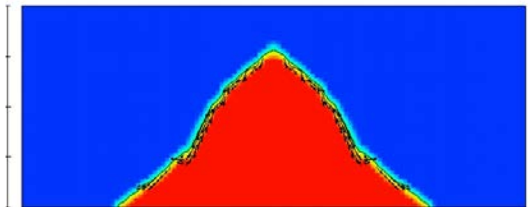


Figure 2c. Time 3.0s.

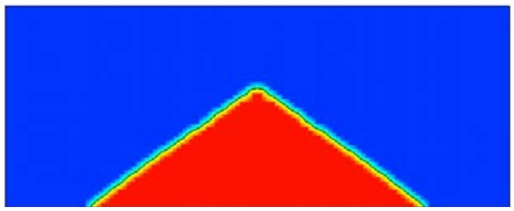


Figure 2d. Time 10.0s.

It can be seen in Fig.2a-2d showing a cross section of the axisymmetric pile that a cone forms as the sand slumps. The slope of the side (estimated by a protractor) is very close to the specified 34° angle of repose. Most importantly, it is observed that only sand close to the surface is seen to move, while sand in the interior of the heap remains stationary. This is a well known feature of granular flow. After about 4.0s the pile is no longer changing shape.

Hour Glass Test

Everyone is familiar with the flow of sand in an hourglass. Sand flowing from a top into a bottom section collects in a conical pile on the bottom. One of the troublesome problems with granular flow is that it may form rat holes or arches at flow constrictions. For example, when granular material is directed into a funnel so it can be put into containers it may form a rigid arch of packed material at the entrance to the narrow funnel tube that completely stops the flow. Alternatively, there may be only a small irregular hole, a rat hole, above the funnel tube that allows flow to occur. More generally, the granular flow will exit the hopper at a more or less constant rate as long as sufficient material remains in the hopper to cover the exit hole.

To look for such behavior in the continuum model a simple two-dimensional hour glass shape was set up with a 1 cm wide tube connecting the top and bottom sections having sides with a slope of 40.5°, which is larger than the angle of repose (34°). The simulation is started with the bottom section empty.

Sand was initialized at its close packing limit of 0.63 volume fraction. Sand at the bottom of the opening to the discharge tube begins to fall under the action of gravity, but nearly all the sand above remain stationary, Fig.3, where the color is the volume fraction of mixture (red being 1.0). In a short time a bubble like region is formed and moves up toward the top surface of the sand. Only flow around the surface of the bubble is seen until the bubble reaches the top then it causes a collapse of the surface. The indentation in the top surface has flow that eventually reduces its sides to the angle of repose. The sand may close off the vertical open channel but should this happen another bubble forms at the bottom to repeat this pattern. The pattern of repeated bubbles rising up through the sand is a realistic phenomena often observed in granular media. At the end of the simulation all but a token of sand remains on the walls of the top section and in the bottom section the sand has stopped flowing with the surface fixed at the angle of repose. Some tiny residual velocities remain near the bottom surface on the order of 1.8e-3cm/s, but are decaying and not affecting the final resting of the sand. This simulation required a CPU time of 43.4min.

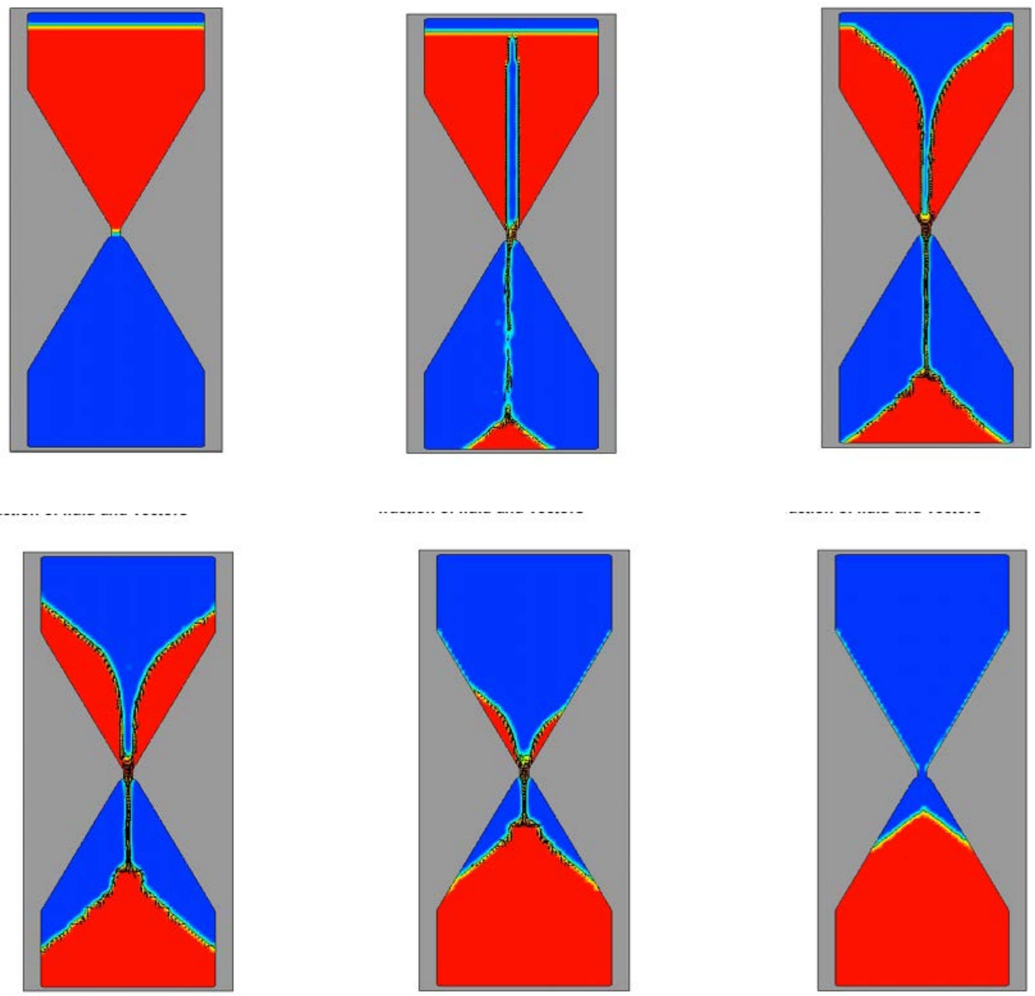


Figure 3a. Two-dimensional hour glass at times 0.0s, 4.0s, 10.0s, 20.0s, 30.0s and 35.0s.

Horizontal Cylindrical Mixer

A common mechanical mixer for granular material consists of a circular cylinder partially filled and set rotating about a horizontal axis. Material in the cylinder tries to rotate as a rigid body with the cylinder, because of wall friction, but the free surface of the material spills downwards when its tilt, with respect to gravity, exceeds the angle of repose.

Figure 4 shows a comparison with an experiment involving colored solids (salt) taken from reference [5]. Initially the uniform solid/air mixture fills 80% of the cylinder and has been colored blue in the region to the left of a vertical center line, while the region on the right side is colored red. The data comparison is after two complete revolutions of the cylinder.

The cylinder radius is 15cm and the density of the solid is 2.6g/cc. A close packing of the solid was assumed initially at its maximum volume fraction of 0.63. An angle of repose of 52° was specified to agree with the value measured in the experiment. The rotation rate was one revolution per minute. For computational simplicity the gravity vector was rotated counter to the cylinder instead of moving the cylinder and its contents.

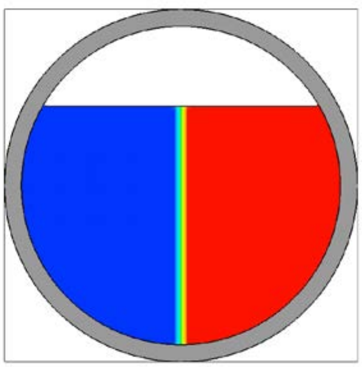


Figure 4a. Simulation of rotating mixer: initial condition 80% filled.

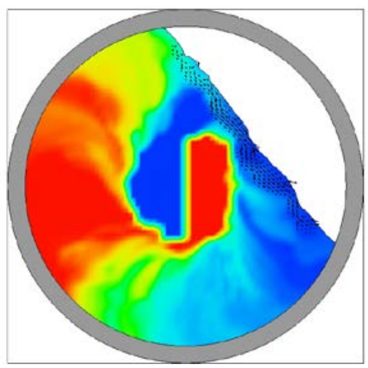


Figure 4b. Simulation of rotating mixer: after 2 revolutions.

Figure 4c. Experimental data for rotating mixer: after 2 revolutions.

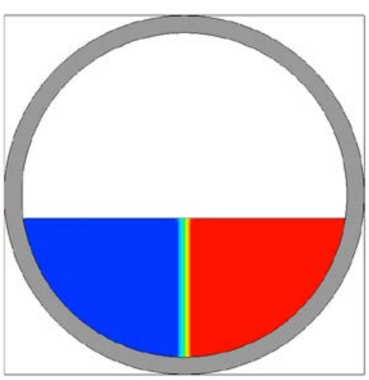


Figure 5a. Simulation of rotating mixer: initial condition 40% filled.

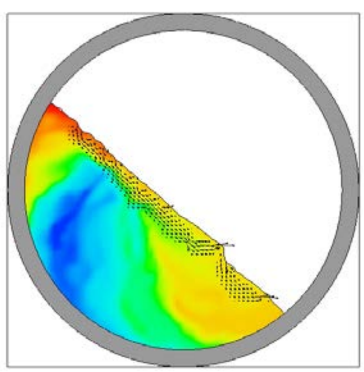


Figure 5b. Simulation of rotating mixer: after 2 revolutions.

Figure 5c. Experimental data for rotating mixer: after 2 revolutions.

An interesting feature of this arrangement is that the central region of the granular material does not mix because the avalanching of the solid particles at the free surface is always a shallow layer that is moving down the slope. This is evident in Fig.4b where black colored velocity vectors can only be seen near the surface. The finite resolution of the computational model causes some numerical mixing (diffusion) of the colors, but overall the comparison between experiment, Fig.4c, and simulation, Fig.4b, is quite good.

The fact that the size of the unmixed central region in the simulation, Fig.4b, is somewhat smaller than in the experiment, Fig.4c, is an indication that the thickness of the layer at the surface that is avalanching is probably a little too large because of the coarseness of the computational grid.

A corresponding comparison between experiment and simulation is shown in Fig.5 where the initial fill fraction of the cylinder was only 40%. Here again the results seem quite good although the experimental picture is not clear enough to allow for a detailed comparison. In this case there is no undisturbed central core.

Horizontal Rectangular Mixer

Another mixer example from reference [5] is similar to the previous one, except that the mixer has a square instead of a cylindrical cross section. All the physical properties are identical to the former example except for the rotation rate, which was 0.5rev/min and the rotation was only 1.25 complete turns. Again, there is very nice correlation between the experiment and simulation as seen in Fig. 6a-c (note that gravity in Fig.6b and 6c is pointed to the right for the quarter turn and that blue was used in the simulation in place of yellow in the experiment).

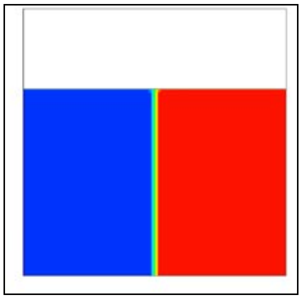


Figure 6. Simulation of rotating rectangular mixer: initial condition of colored material.

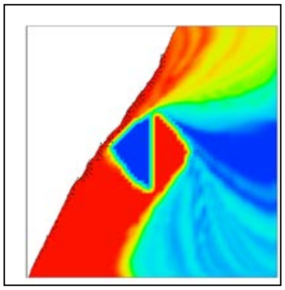


Figure 6b. Simulation of rotating rectangular mixer after 1.25 revolutions.

Figure 6c. Experimental data for rotating rectangular mixer: after 1.25 revolutions.

Illustrative Examples Part II: Solid/Liquid Mixtures

Granular material contained in a liquid, or slurry, arises in many industrial activities. Similarly, debris flows arising in runoff in mountain valleys, mud flows and other environmental events involving solid material in water are equally important granular flows. In this section several examples are presented showing the use the granular flow model for mixtures of solid/liquid materials. One of these examples, a slurry flume, illustrates the importance of a dispersive pressure feature included in the model.

Hydrostatic Test

The simplest test illustrating basic features of a granular flow is the settling of solids dispersed in a column of fluid, in which there is no mean flow. This case is to be compared with the corresponding solid/gas mixture test shown in Fig.1. Here the mixture is somewhat typical of concrete consisting of water and solid having a density of 2.6gm/cc and grain diameter of 0.5cm. Initially the solid fraction is uniform at 0.3125. Figure 7 shows how the solid settles out leaving a region of pure water at the top, with the initial free surface (i.e., interface with air) unchanged.

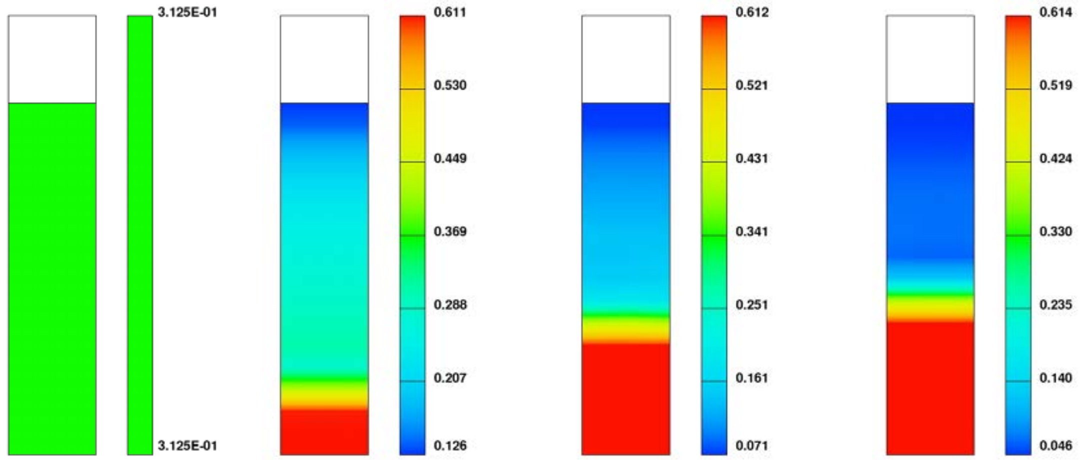


Figure 7a. Color indicates solid fraction in granular mixture. Initial solid fraction is 0.3125. Subsequent frames are at times 1s, 5s and 20s. At late time there is a region of nearly pure water (solid fraction less than 5%) separating air from the packed solid at the bottom of the column. Also note that there is no net volume change in the water/solid mixture as the solid settles; compare with Fig.1 for an air/solid mixture.

Slump of Cylindrical Column

The Heap Test shown in Fig.2 can be repeated for a mixture of solid and liquid. Here the mixture is sand in water in which the sand grain diameter is 0.5cm a large value with gravel-like concrete in mind. This is probably too large in view of the 0.25cm grid size that was used for the simulation but is a good reminder that we are not dealing with discrete particles, but a continuum description of granular media. The cylinder is 10cm in diameter and 10cm in height. Initially it is at rest on a horizontal surface and then slumps because of gravity.

When the initial solid fraction of the cylinder is at its maximum packing fraction of 0.63 the slump shown in Fig. 8a resembles that for sand column forming a cone with a surface inclined at the specified 34° angle of repose. However, if the initial solid fraction is reduced to 0.55 then the mixture slumps much further because the water carries the sand before it has time to reach sufficient concentration for jamming. This leaves a highly packed region with lower height and wider footprint, Fig.8b.

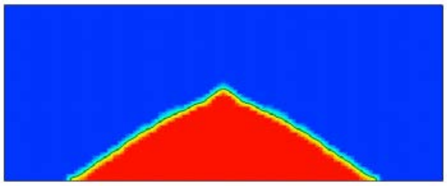


Figure 8a. Final configuration of a highly packed cylinder of gravel and water.

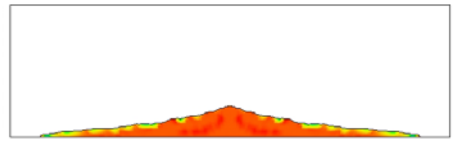


Figure 8b. Final configuration for a cylinder of gravel/water mixture with an initial low packing fraction 0.55.

Slurry Flume: Loose Bed

A slurry flume consisting of a sloped channel 6m in length and 0.2m in width and having a gate at the top end allowing a prescribed mixture of solid particles and water to enter at a specified flow rate has been used by Larcher, et al [7] to obtain a highly useful set of experimental data. The solid material consisted of pellets of extruded PVC with an equivalent spherical diameter of 0.0035m and density 1540kg/m^3.

Numerous experiments were performed that identified several flow regimes. Steady flow conditions in which there is a loose packed bed of non-flowing solid particles on the channel floor, but solid material in the remaining liquid up to the free surface are called loose bed debris flows. Experimental Run No. 85, a loose bed debris flow, has been selected for this validation test: the channel slope is 6.2°, the volume flow rate at the inlet gate is 4.7e-3m^3/s and the mean solid volume fraction at the gate is 0.35.

A simulation was run to t=50s; a true steady-state condition does not develop because of a slow buildup of deposited solid near the entrance. However, near the outlet the flow has steadied out and remains steady for the last 10s of the simulation. The experimenters comment on the requirement of “tuning” their experiments in loose bed cases to reach a critical balance between entrainment and deposition of solid to achieve a steady flow. This type of balance was not achieved in the simulations. A plot of the entire 6m flume is displayed in Fig.9 at t=50s, where the uniform region at the outlet end of the channel can be clearly seen. This shows the solid fraction distribution with a high concentration at the bottom of the channel where a packed bed has formed that is no longer able to flow, in agreement with the experimental observations.

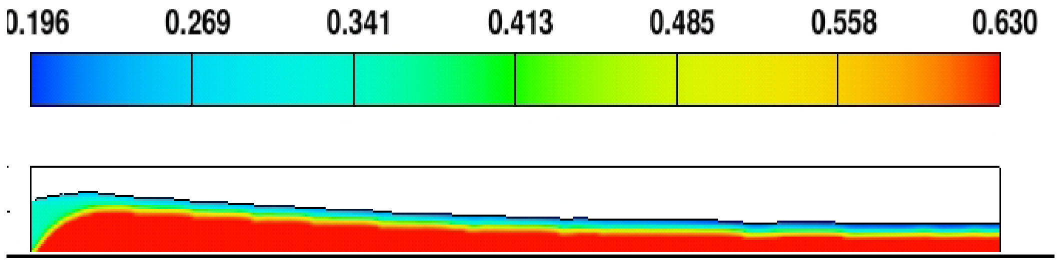


Figure 9. Cross section of flume simulation for run #85. Color is solid fraction.

Comparison of the computed and experimental data are presented in Fig.10, with Fig.10a comparing the vertical distribution of the velocity component (parallel with the channel floor) and Fig.10b the vertical distribution of the solid fraction. In the experimental data for solid fraction it is reported that the maximum values in the packed bed are said to be only estimates because the method used to compute them broke down. The experimenters assumed a maximum solid fraction of 0.69 at close packing while the simulations used a value of 0.63. The plot in Fig.10b uses the simulation value of 0.63 for the experimental data, which is well within the measured accuracy. In any case, videos of the flow clearly indicated that in this region there was no flow of the solid material, which was also a feature in the simulation.

Additionally, as an immature loose bed flow, there is a thin region, sometimes called a sheet flow, on the top surface that is mostly water and was not included in the experimental data, but can be seen in Fig.4b of reference [7]. This top region appears to extend another 1.5cm above the 5.5cm layer in which they provide data. The simulations include this top region (although it is not plotted) and indicate a flow velocity somewhat larger than 1.0cm/s and a solid fraction that is at or below 0.2.

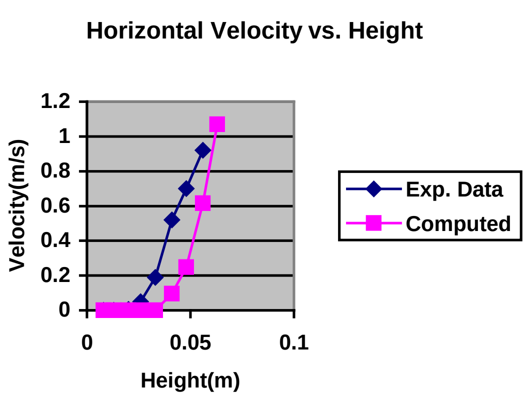


Figure 10a. Comparison of experimental Run #85 from [7] and computed data for horizontal velocity as a function of height above channel floor.

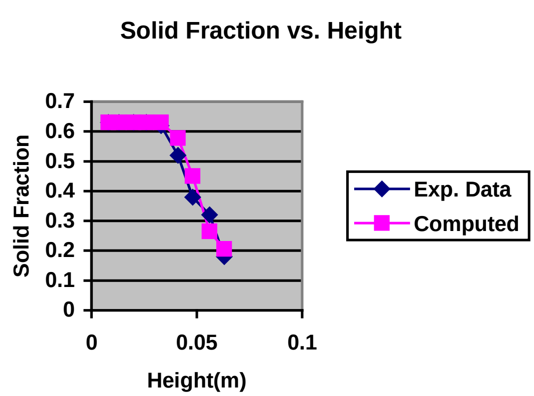


Figure 10b. Comparison of experimental Run #85 from [7] and computed data for solid fraction as a function of height above channel floor.

The agreement between computations and observations is quite good, although it might be questioned as to why the computation did not attain a steady state. One answer to this is that no information was given about the entrance to the experimental channel and how the water and solid were mixed at that location. It is only known that some sort of “tuning” was needed to get steady results in the experiments that involved a packed bed of solid.

Slurry Flume: Solid Bed (Dispersive Pressure Example)

For a second example of the slurry flume, a “solid bed” regime, Run No. 46, was selected as a validation test. In this regime no packed bed develops (“solid bed” refers to the bed of the experimental channel) and, most crucially, the solid fraction in the flow remains well below a level where the solid grains would be in contact. For Run No. 46 the channel slope was increased to 23°, the flow rate was 12.4e-3m^3/s and the initial solid volume fraction was 0.49. For this run a steady-state was quickly reached in the simulation.

This test is crucial because it demonstrated that an early version of the granular flow model that did not include a dispersive pressure was unable to represent this situation. To explain why this was so it is necessary to first understand where the dispersive pressure comes from.

Bagnold [6] introduced the notion of dispersive pressure in connection with the flow of sand and air. At high solid fraction concentrations, the principal viscous mechanism in a solid/fluid mixture arises from collisions between grains, which are very effective in transferring tangential momentum in a shearing flow. When particle collide, however, they deflect one another and in addition to exchanging tangential momentum also tend to move away from each other in a direction normal to the shear. This tendency for grains to move apart and opening up a larger space between them is also referred to as Reynolds dilatation when it occurs during an initial application of a shear flow. Bagnold recognized that in general a collision generated shear stress should always have a corresponding normal stress, which he called the dispersion pressure, because it tends to disperse the solid grains. The relation between the two is expressed simply as,

$$\boldsymbol{T} = \tan(\theta_{friction})\boldsymbol{P} , \qquad (1)$$

where $\boldsymbol{T}$ is the tangential collision stress and $\boldsymbol{P}$ the corresponding dispersive or normal stress. The friction angle $\theta_{friction}$ is a property of the granular material that is associated with frictional stresses acting between grains. More generally, it is related to the concept of static friction.

Imagine a box of sand that is tilted allowing some sand to flow out one side. After a short time the sand remaining in the box will come to rest with its surface having a slope whose angle is called the angle of repose. If the box is then slowly tilted a little more the sand will not flow until it reaches a somewhat larger slope having an angle referred to as the angle of motion or angle of friction. From a physical point of view when the sand is resting at the angle of repose the surface sand grains have settled into small depressions between neighboring grains. In order for them to move again it is necessary to supply a stress that is large enough to raise them out of the depressions where they can again flow down the slope. This extra stress needed for setting the surface grains in motion has both a normal and tangential component with respect to the surface, and the relation between them is expressed by Eq.1. From this relation is can be seen that increasing the friction angle decreases the dispersive (normal) pressure.

The friction angle found from experiments is roughly about 2 to 8 degrees larger than the angle of repose. It is generally larger when the grains are irregular and also larger when surrounded by liquid rather than gas, because of additional fluid viscosity influences. Typical values for various types of sand and gravel range from 32° to 40°.

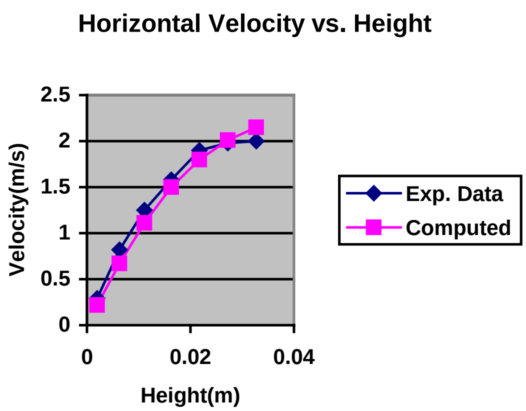


Figure 11a. Comparison of experimental Run #46 from [7] and computed data for horizontal velocity as function of height above channel floor.

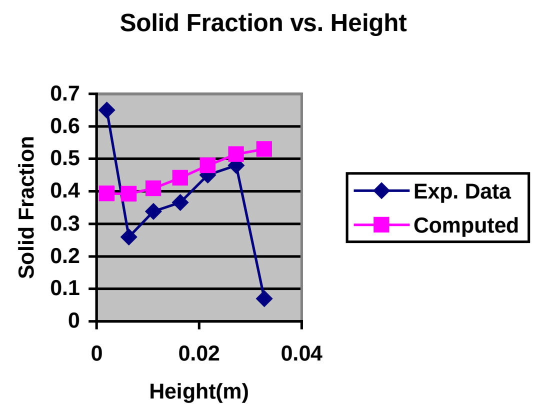


Figure 11b. Comparison of experimental Run #46 from [7] and computed data for solid fraction as function of height above channel floor.

The importance of the dispersive pressure can be appreciated by looking at the experimental data for solid fraction versus depth in the steady-state flow of Run #46 near the outlet of the flume, Fig.11b. The maximum solid fraction at the bottom of the channel is said to be a “measurement outlier”, while the remaining data points are assigned a error range of a little more that ±0.1, which puts nearly all the computed values within the data range

The significance of this data is that the solid fraction remains well below a value where grains would have sufficient contact to prevent them from settling. There is no other known mechanism to prevent settling in this region than a shear-created dispersive pressure (i.e., normal stress associated with grain collisions). For this reason, the experimental data for Run #46 provides an important test to show that this feature of granular flow has been captured in the present model.

To further emphasize the influence of the dispersive pressure, Fig.12, upper frame, shows the first 0.8m portion of the flume profile when flowing at steady conditions (the color indicates solid fraction). When this simulation is restarted from its steady state, but with the dispersive pressure removed, the solution quickly deteriorates as can be seen at 0.2s later in Fig.12, lower frame. There is a disruption at the inlet (sending some fluid to the top of the channel), but most importantly, the solid fraction distribution has been inverted with the maximum now at the channel bottom and the minimum at the top, which is consistent with gravitational settling when there is no dispersive pressure to resist settling.

The 0.2s time interval between plots in Fig.12 seems short, but it is because the asymptotic sinking velocity of a single grain in water is about 22cm/s and the slurry is only 3cm deep, so that a grain can sink to the bottom in about half that time interval.

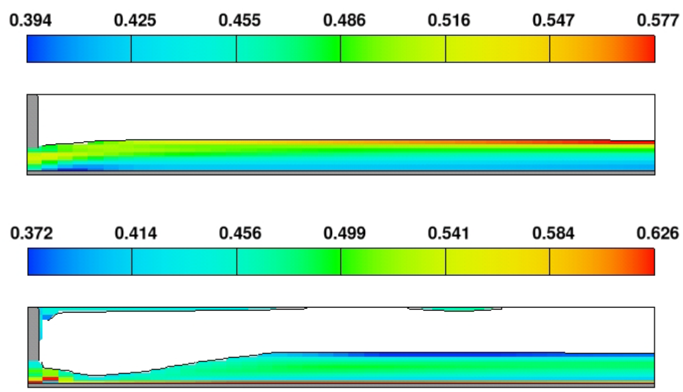


Figure 12. Comparison of flow in first 0.8m of flume for Run #46 from [7] with dispersive pressure (top) at steady conditions and after another 0.2s (bottom) without a dispersive pressure. Color indicates solid fraction.

Use of a dispersive pressure requires the specification of a friction angle. In Larcher, *et al* [7] a friction angles of 31° and 35° are quoted. For the simulations it was found that a friction angle of 48° lowers the dispersive pressure and gives the best results. Reasons for this difference are unknown, although it might be mentioned that the experimental data given for tangential and normal stresses are peculiar. For example, there are smooth tangential stresses extending through packed regions of the flow where there is a zero tangential velocity, and there are large normal stresses where the tangential stress has gone to zero even though these stresses should be proportional to one another.

One further comment about the dispersive pressures is worth mentioning. In most cases the influence of a dispersive pressure is more likely to have a greater influence in solid/liquid mixtures than in solid/gas mixtures because it does not have to compete with the greater negative buoyancy of solid grains in air compared to that in water. The Rotating Square Mixer in Fig.6 offers a good example; including a dispersive pressure or not results in no significant difference in the computed results.

Illustrative Examples Part III: Qualitative Results

A common test for an avalanche-type behavior that is observed in materials having a yield stress is to place a heap of material on a flat surface then tilt that surface until the heap begins to flow. Typically an avalanche is observed when the material has some sort of granular structure that can jam into a rigid state, but that will begin to flow when a stress applied to the material exceeds a critical value. This is also a classic description of sand that exhibits an angle of repose. A sand surface having an angle, with respect to the horizontal, that is below the angle of repose does not flow, but when the angle is increased above the angle of repose flow is initiated at the surface and often evolves from a few grains into a much larger flow, i.e., an avalanche of sand.

A nice example of an avalanche flow from a heap is given in Coussot, et al [8]. In this case the material is a clay suspension but, as the authors say, the results are remarkable similar to the flow of a granular material. To show this similarity two simulations have been done, one simulation is for sand in air and the other for sand in water. In both cases a spherical cap of the granular mixture is placed on a flat surface tilted 12° above the horizontal. The radius of the cap is 29.86cm and has a maximum height above the solid surface of 4cm. The initial solid volume fraction is 0.61 just above a jamming solid fraction of 0.60. The grains have diameter 0.025cm and a density 2.6gm/cc. The angle of repose is specified as 34°.

Figure 13 shows the initial heap (a) and the flow after a period of 5s (b). For comparison the experimental result for the clay suspension from [8] is shown in frame (c). The times and scales are different because of the different material types, but the similarity of the results is evident. In particular, the crescent shape of the collapsing sand in the heap and the narrow flow leaving the bottom edge of the heap.

Something not seen in the experimental result is the changing solid fraction observed in the sand, Fig.13b. This does not happen in the clay because it is a suspension and there is no settling of the solids.

For comparison it is instructive to repeat the simulation with water surrounding the sand grains instead of air. With no other changes than to increase the friction angle for dispersive pressure (from 18° to 45° as was needed in the flume tests) the results are shown in Fig.14a at 2s and in Fig.14b at 5s.

There is a clear difference between this case with water and the former case with air. The flow is spreading more to the sides as it flows down the slope. Sand is settling out of the upper layers of the mixture resulting in a lower mixture viscosity, which allows a faster flow in the lower-solid-fraction material, allowing flow down the slide, as well as to the sides

As can be seen in Fig.14b, the solid fraction is decreasing uniformly further down the slide. It is easy to visualize the sand settling out leaving the water to flow downward more freely.

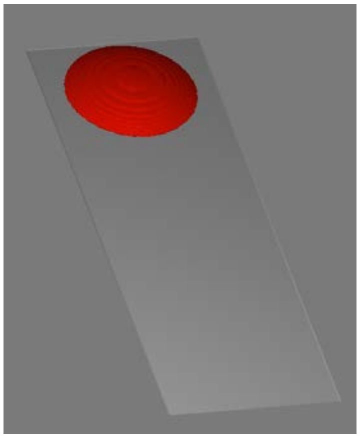


Figure 13a. Initial heap of sand/air on a tilted slide.

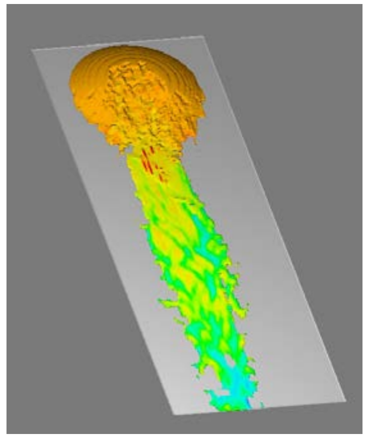


Figure 13b. Flow of sand after 5s. Color is solid fraction. The plotted surface is fluid fraction 0.2.

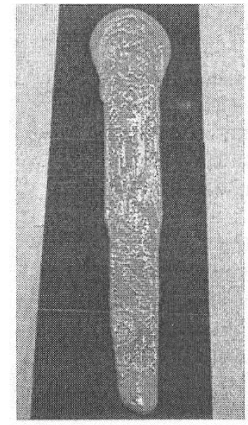


Figure 13c. Flow of clay suspension heap from [8].

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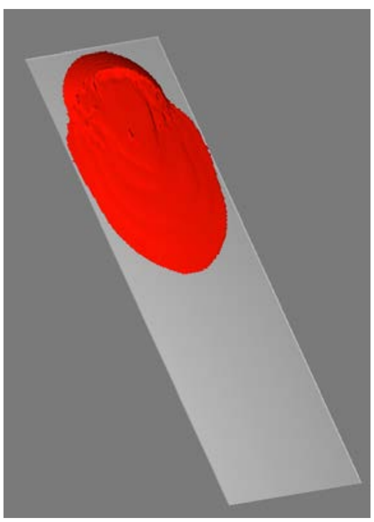

Figure 14a. Heap flow on slide for sand/water mixture after 2s.

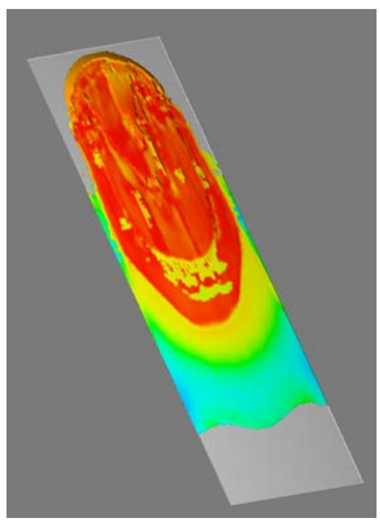

Figure 14b. Flow after 5s. Color is solid fraction and shows how settling sand has allowed the mixture to flow further down the slide.

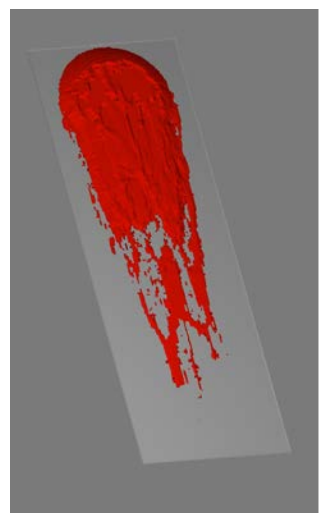

Figure 14c. Flow after 5s when the initial solid fraction has been set at its maximum value. Plotted surface is fluid fraction of 0.2.

Finally, in Fig.14c this simulation was repeated with the initial condition replaced by a mixture with the maximum possible solid fraction of 0.63. Since the solid is at its maximum concentration no settling is possible and the slump of the heap retains a constant solid fraction and has a result that appears more similar to the case of sand in air shown in Fig.13b. In all the cases with water the crescent shape of the collapsing heap is very evident.

Possible Extensions to the Continuum Model

There are several possible extensions to the basic granular media continuum model that might improve its realism for a wider range of applications.

Packing Limits

For example, it is likely that the maximum close packing limit could vary as a function of compressive stress applied to the sand-air mixture. The fluidity of packed solid could also be directly related to its packed density, i.e., fluidity decreasing with increasing packed density.

Binders and Cohesive Forces

Many types of granular material exhibit cohesive forces between grains, for example moist sand. This issue has not been addressed in the current model. There are, however, several ways in which the effects of cohesive forces between individual grains might be included in the model.

One example is in the threshold velocity for packing. Certainly when the solid fraction exceeds the mechanical jamming limit grains are experiencing some contact and this is the region where the packing drag force is applied. An increase in the threshold velocity needed to remove particles from a packed bed when there are cohesive forces tending to hold the particles together is an obvious change that can be made by using an input variable that is a multiplier on the threshold velocity. A similar input variable is available to increase the drag resistance to flow in partially packed regions. Also, the angle of repose would be expected to be greater when cohesive forces are acting, and it might be necessary to account for the thickness of coatings when setting the close packing volume fraction. All these options are possible in the present model, but have not been explored because of a lack of data with which to test them.

Multiple Particle Sizes

A particularly desirable extension would be the introduction of a mixture of different grain sizes. Even a mixture of consisting of only two sizes would be highly useful if the sizes could be separated, with the larger above the smaller, when the mixture is shaken at high packing densities. Bagnold [6] has suggested that the dispersive pressure may be one cause for the separation of different grain sizes when a mixture is sheared. Other mechanisms can also be imagined. More research and development is needed for this extension to be made.

Summary Comments

A continuum fluid model for granular material has been described and illustrated with numerous examples. Granular material in either a gas or liquid fluid is permitted. The model has been constructed using empirical relations obtained from the cumulative work of numerous researchers over many years starting with the ground-breaking work of Bagnold. In particular, relations for the mixture viscosity and for a threshold velocity associated with packing are key elements of the new model. Additional relations have been developed for packing resistance, dispersive pressure, angle of repose and for the two-phase flow necessary for changes in granular density.

While it has been emphasized that the model has some limitations, such as not allowing for a detailed flow of gas or liquid through packed solids, it nevertheless produces results with realistic features. Applications to a variety of common granular processes show good qualitative agreement with the experimental data available.

Finally, it has also been shown that the present continuum model reproduces many characteristic features of granular flows that do not occur in pure fluids. For instance, there can be regions of packed solid where flow is prohibited entirely or is only exhibited along free surfaces having a slope exceeding the angle of repose. Pressures in fully packed regions also revert to the external continuum pressure because the solid is no longer part of a fluid mixture when it is packed and able to support itself by contact forces. Solid grains can settle when disturbed, but only to a maximum specified solid fraction. Dispersive pressures are included that can keep solids from packing in regions having a sufficiently large shear flow. All these features support the fundamental construction and utility of the granular flow model.

References

1. Flow Science, Inc. Technical Note TN77 (2007).

2. Flow Science, Inc. Technical Note TN83 (2009).

3. Bagnold, R.A., The Physics of Blown Sand and Desert Dunes (1941); reprinted Dover Publ., Mineola, N.Y. (2005).

4. Mih, Walter C., “High Concentration Granular Shear Flow”, J. Hydraulic Res., **37**, p229 (1999).

5. Metcalfe, G., Shinbrot, T., McCarthy, J.J. and Ottino, J.M., “Avalanche Mixing of Granular Solids,” Nature, **374**, p39 (1995).

6. Bagnold, R.A., “The Shearing and Dilatation of Dry Sand and the ‘Singing’ Mechanism”, Proc. Roy. Soc. A, **295**, p219 (1966).

7. Larcher, M., Fraccarollo, L., Armanini, A. and Capart, H., “Set of Measurement Data from Flume Experiments on Steady Uniform Debris Flows,” J. Hydraulic Res., **45** Extra Issue, p59 (2007).

8. Coussot, P., Nguyen, Q.D., Huynh, H.T. and Bonn, D., “Avalanche Behavior in Yield Stress Fluids,” Phy. Rev.Let., 88, pp175501-1 (2002).