

# Numerical Investigation of Wave Interaction with Double Vertical Slotted Walls

H. Ahmed, A. Schlenkhoff

**Abstract**—Recently, permeable breakwaters have been suggested to overcome the disadvantages of fully protection breakwaters. These protection structures have minor impacts on the coastal environment and neighboring beaches where they provide a more economical protection from waves and currents. For regular waves, a numerical model is used (FLOW-3D, VOF) to investigate the hydraulic performance of a permeable breakwater. The model of permeable breakwater consists of a pair of identical vertical slotted walls with an impermeable upper and lower part, where the draft is a decimal multiple of the total depth. The middle part is permeable with a porosity of 50%. The second barrier is located at distant of 0.5 and 1.5 of the water depth from the first one. The numerical model is validated by comparisons with previous laboratory data and semi-analytical results of the same model. A good agreement between the numerical results and both laboratory data and semi-analytical results has been shown and the results indicate the applicability of the numerical model to reproduce most of the important features of the interaction. Through the numerical investigation, the friction factor of the model is carefully discussed.

**Keywords**—Coastal structures, permeable breakwater, slotted wall, numerical model, energy dissipation coefficient.

## I. INTRODUCTION

SINCE a few decades ago, permeable breakwaters are used to provide environmental and economic protection from undesirable waves and currents inside the ports. These types are partially protection breakwaters and have been suggested to overcome the environmental disadvantages of fully protection breakwaters and to reduce the transmission of waves and currents inside the harbors and to protect beaches and coast lines as well. Those structures have been already implemented in form of pile breakwaters, which are formed from a series of piles, placed in rows. Such breakwaters exist e.g. at Hanstholm (Denmark), Marsa el Brega (Libya), Osaka (Japan) and Pass Christian Mississippi (USA), [23] and implemented in form of curtainwall-pile breakwaters at Yeoh port (south coast of Korea), [22].

In some instances, a single permeable barrier, such as a slotted vertical barrier, may be used. Suh et al. [21], [22] discovered theoretically and experimentally the hydrodynamic characteristics of a curtain-wall-pile breakwater. The model

consisted of an impermeable upper part in form of a vertical wall and the lower part consisted of an array of vertical square and circular piles.

Laju et al. [12] investigated the pile supported skirt breakwater. The breakwater model consisted of an impermeable wave barrier near the free surface supported on steel or concrete piles. Rageh and Koraim [18] reported the hydraulic performance of a vertical wall with horizontal slots. The upper part was impermeable but the lower part of model was horizontal slots. Ahmed et al. [2] have investigated the hydrodynamic characteristics of a vertical slotted wall; which can be used as a breakwater. The model consists of an impermeable upper and lower part; the draft of them is a proportion of the total depth and the middle part is permeable with a porosity of 50%.

For more protection and more dissipation of energy a pair of permeable barriers may be preferred. Isaacson et al. [7] suggested using a pair of thin vertical barriers extending from the water surface to some distance above the seabed. This study carried out through theoretical and experimental investigation. The interaction between the impermeable upper part and the slotted lower part was investigated experimentally by Sundar and Subbarao [23]. The model basically consisted of two parts: the top portion with quadrant front face, and the piled structure consisted of two units, each with two rows of staggered piles at the bottom. Koraim et al. [10] investigated the hydrodynamic characteristics of double permeable breakwater under regular waves. The model consisted of double walls with horizontal slots.

Thus, the prediction of wave interactions with a permeable wall or multi rows of it is also of interest. A primary feature of such interactions is that wave energy is absorbed within the permeable barriers. Several authors have investigated the hydraulic performance of permeable breakwaters theoretically and experimentally. Theoretical solutions have been developed on the basis of the boundary element method, reported by [15]. The Eigenfunction expansion method has been utilized as a theoretical approach for linear wave by [1], [7] and [8], [10], [11] and for nonlinear waves (Stokes second-order wave) by [3].

CFD applications are common practice in all sectors of engineering, and they are becoming increasingly important in maritime and coastal engineering. Numerical simulations have been performed by using a commercial CFD code (FLOW-3D, Flow Science Inc.). The finite volume method can solve the three-dimensional Reynolds-Averaged Navier-Stokes (RANS) equations. The wave free surface can be determined by using the Volume of Fluid (VOF) method [5]. Numerical solutions

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for wave interaction with structures have been thoroughly and practicably tested. Interesting examples for wave interaction with porous and submerged structure are reported e.g. in [6], [13], [14]. Lin et al. [16], Karim et al. [9]. Schlenkhoff et al. [19] investigated regular wave interaction using a numerical model of (FLOW-3D, VOF) with a vertical slotted wall. The efficiency of these numerical models have been improved in order to investigate wave interaction with structures, at distinguish low costs when compared with laboratory studies.

In the present study, a numerical model is used (FLOW-3D, VOF) to investigate the hydraulic performance of a permeable breakwater in form of a double vertical slotted wall for linear waves. In this study, we focus on wave transmission and wave energy. The friction and shear stress have a great influence on the reflection and transmission and is carefully discussed. Comparisons of numerical results with available theoretical and experimental data of Ahmed [4] are conducted to validate the numerical model.

## II. THEORETICAL AND NUMERICAL CONSIDERATIONS

The model consists of a pair of identical vertical slotted walls. The first wall is located in the seaward direction at a distance  $(-\lambda)$  and the second unit is located in the shoreward direction at a distant  $(\lambda)$  from the origin point which is located at the water surface as shown in Fig. 1. Both walls have an impermeable upper and lower part, where the draft is a decimal multiple of the total depth and the middle part is permeable with a porosity of 50 %.

The present study focuses and compares the experimental setup which has been used in the previous study by the author in 2011 [4].

The physical model test has been done previously in the hydraulic lab of the University of Wuppertal. A small wave flume has been used ( $L = 25$  m,  $B = 0,3$  m and water depth  $d = 0,3$  m). The model scaled approximately with  $M = 1$  to 10 to 1 to 20. The reflection coefficient is then calculated by the three-probe method of Mansard and Funk [17]. The selected data are converted into frequency domain by Fast Fourier Transformation. Finally, the spectrum of the incident, transmitted and reflected wave height are calculated. Thereby, the reflection coefficient 'CR' is calculated from extracted wave profiles by:

$$CR = h_r h_i^{-1} \quad (1)$$

where  $h_r$  and  $h_i$  are the reflected and incident wave heights, respectively. The transmission coefficient 'CT' is calculated directly from the wave transmitted profile by:

$$CT = h_t h_i^{-1} \quad (2)$$

where  $h_t$  is the transmitted wave height. The energy dissipation coefficient 'CE' is given by:

$$CE = 1 - CR^2 - CT^2 \quad (3)$$

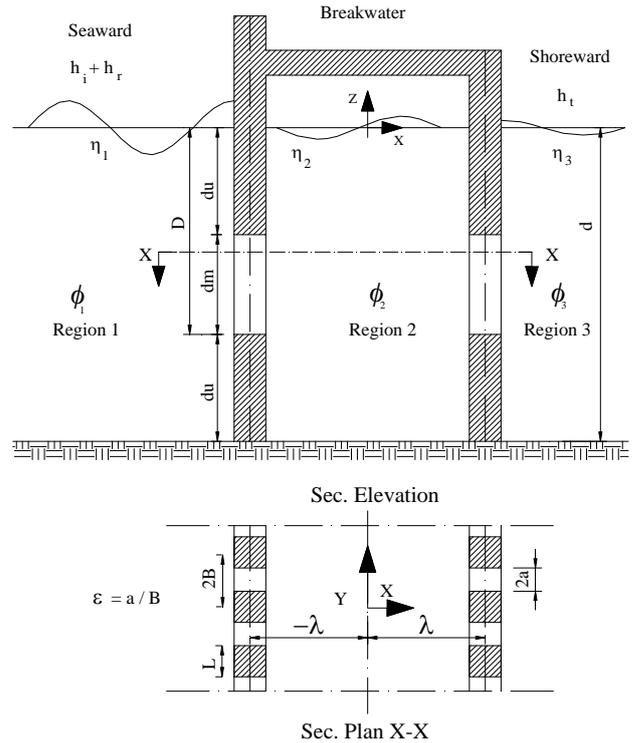


Fig. 1 Definition sketch of double vertical slotted wall breakwater

The previous theoretical and semi-analytical investigation has used the Eigen function expansion method and has utilized the boundary condition at the surface of each wall. The domain is subdivided into three regions and the two-dimensional potential functions are denoted as  $\phi_1$ ,  $\phi_2$  and  $\phi_3$  in regions 1, 2 and 3 respectively. The permeable boundary condition along the structure has been developed on the basis of the formulation of Sollitt and Cross [20] and as adopted by Yu [24] for a thin vertical barrier. This may be given by:

$$\frac{\partial \phi_1(x)}{\partial x} = \frac{\partial \phi_2(x)}{\partial x} = iG^-(\phi_1(x) - \phi_2(x)) \quad (4)$$

at  $x = -\lambda$  for  $-D \leq z \leq -du$

$$\frac{\partial \phi_2(x)}{\partial x} = \frac{\partial \phi_3(x)}{\partial x} = iG^-(\phi_2(x) - \phi_3(x)) \quad (5)$$

at  $x = \lambda$  for  $-D \leq z \leq -du$

The proportional constant  $G^- = \frac{G}{b}$ ,  $G$  is called the permeability parameter and is expressed by:  $G = \frac{\epsilon}{f - is}$

where  $\epsilon$  is the porosity of the structure given by the dimension and spacing of the piles,  $f$  is the friction factor and  $S$  is the inertia coefficient and given by  $s = 1 + cm \left( \frac{1 - \epsilon}{\epsilon} \right)$

where  $cm$  is an added mass coefficient.

Eigenfunction expansion solved the velocity potential in a series of infinite number of solutions as follow:

$$\phi_1(x) = \phi_i + \sum_{m=0}^{\infty} A_{1m} \cos[\mu_m(d+z)] \exp(\mu_m(x+\lambda))$$

at  $x \leq -\lambda$  (6)

$$\phi_2(x) = \sum_{m=0}^{\infty} A_{2m} \cos[\mu_m(d+z)] \exp(-\mu_m(x+\lambda)) +$$

$$\sum_{m=0}^{\infty} A_{3m} \cos[\mu_m(d+z)] \exp(\mu_m(x-\lambda))$$

at  $-\lambda \leq x \leq \lambda$  (7)

and

$$\phi_3(x) = \sum_{m=0}^{\infty} A_{4m} \cos[\mu_m(d+z)] \exp(-\mu_m(x-\lambda))$$

at  $x \geq \lambda$  (8)

Applying the matching conditions (combining (6), (7) and (8) with (3) and (4) at the breakwater; the coefficients  $A_{1m}$ ,  $A_{2m}$ ,  $A_{3m}$  and  $A_{4m}$  can be determined by the following matrix equation:

$$\begin{bmatrix} \sum_{m=0}^{\infty} C_{11}^{(mn)} & \sum_{m=0}^{\infty} C_{12}^{(mn)} & \sum_{m=0}^{\infty} C_{13}^{(mn)} & \sum_{m=0}^{\infty} C_{14}^{(mn)} \\ \sum_{m=0}^{\infty} C_{21}^{(mn)} & \sum_{m=0}^{\infty} C_{22}^{(mn)} & \sum_{m=0}^{\infty} C_{23}^{(mn)} & \sum_{m=0}^{\infty} C_{24}^{(mn)} \\ \sum_{m=0}^{\infty} C_{31}^{(mn)} & \sum_{m=0}^{\infty} C_{32}^{(mn)} & \sum_{m=0}^{\infty} C_{33}^{(mn)} & \sum_{m=0}^{\infty} C_{34}^{(mn)} \\ \sum_{m=0}^{\infty} C_{41}^{(mn)} & \sum_{m=0}^{\infty} C_{42}^{(mn)} & \sum_{m=0}^{\infty} C_{43}^{(mn)} & \sum_{m=0}^{\infty} C_{44}^{(mn)} \end{bmatrix} \begin{bmatrix} A_{1m} \\ A_{2m} \\ A_{3m} \\ A_{4m} \end{bmatrix} = \begin{bmatrix} b_{1n} \\ b_{2n} \\ b_{3n} \\ b_{4n} \end{bmatrix}$$

For  $n = 1, 2, 3, \dots, \infty$  (9)

The solution of (9) is truncated to a finite number of terms ( $N$ ), and thus becomes a complex matrix equation of rank  $4N$ , which can be solved for the first  $N$  unknown values of each set of coefficients  $A_{1m}$ ,  $A_{2m}$ ,  $A_{3m}$  and  $A_{4m}$ . The real reflection and transmission coefficients, denoted as ( $CR$ ,  $CT$ ) respectively, are given in terms of  $A_{1m}$  and  $A_{4m}$  by:

$$CR = |A_{10}| \quad \text{and} \quad CT = |A_{40}|.$$

The energy dissipation coefficient can be determined by the (3).

Numerical CFD simulations with FLOW-3D have been used to investigate the wave interaction with the permeable structure. In order to get a good compromise between precision/accuracy and computation time, two independent meshes with different cell sizes are used. Mesh cells are sized by 1 cm in each direction for waves of frequencies of  $F = 1.25; 1.00; 0.75$  and  $0.50\text{Hz}$ . Mesh cell size of 0.5 cm are used for waves of frequencies  $F = 1.50; 1.75$  and  $2.00\text{Hz}$ . The time window for analyzing the wave height is carefully selected

according to the wave length and is adjusted to avoid any reflection from the flume end or the wave paddle. Three-dimensional solid barriers are modeled as collections of blocked volumes and surfaces. The numerical model within FLOW-3D fully represents the physical model's geometrical and hydraulic boundary conditions.

### III. RESULTS AND DISCUSSION

#### A. Validation of the Numerical Model

The wave interaction is summarized by the reflection, transmission and energy dissipation coefficients as a function of  $kd$  ( $k = \text{wave number}$ , respectively  $\text{wave length } L$  given by:  $k = 2\pi/L$ ), which have been determined [see (1)-(3)]. The middle part of the structure (model) investigated is permeable with a porosity  $\mathcal{E} = 50\%$ . The water depth is  $d = 0.3$  m, the thickness of breakwater  $b = 2.5$  cm,  $h_v/L = 0.025$ , the upper and lower parts are impermeable. Various permeability drafts have been simulated as a proportion of water depth,

$$dm = 0.2d, 0.6d \text{ in Fig. 2 and } dm = 0.4d \text{ and } 0.8d \text{ in Fig. 3.}$$

The draft of the upper and lower parts changed according to  $dm$ . The friction and the added mass coefficient are taken as  $f = 2$ , and  $cm = 0$  throughout this comparison. The chamber width vary as a proportion of the water depth  $2\lambda = 0.5d$ , and  $1.5d$ .

The numerical model has been validated by comparisons with similar laboratory tests and semi-analytical results based on the Eigen function expansion method of Ahmed [4]. As expected, the reflection coefficient decreases with  $dm/d$ , while the transmission coefficient increase with  $dm/d$  and the reflection coefficient,  $CR$ , increases with increasing  $kd$  at fixed  $dm$  and increases with decreasing  $dm$  for a fixed  $kd$ . The transmission coefficient,  $CT$  follows the opposite trend. The numerical model confirms that, using of the second barrier dissipates to up more than 80%, especially for intermediate wave length. The figures show, that the numerical results are fundamentally satisfying the expectations. Hence, the numerical model is able to describe the wave interaction of a linear wave with double vertical slotted walls. Generally, the numerical model is capable to adequately reproduce most of the important features of the experimental and theoretical results.

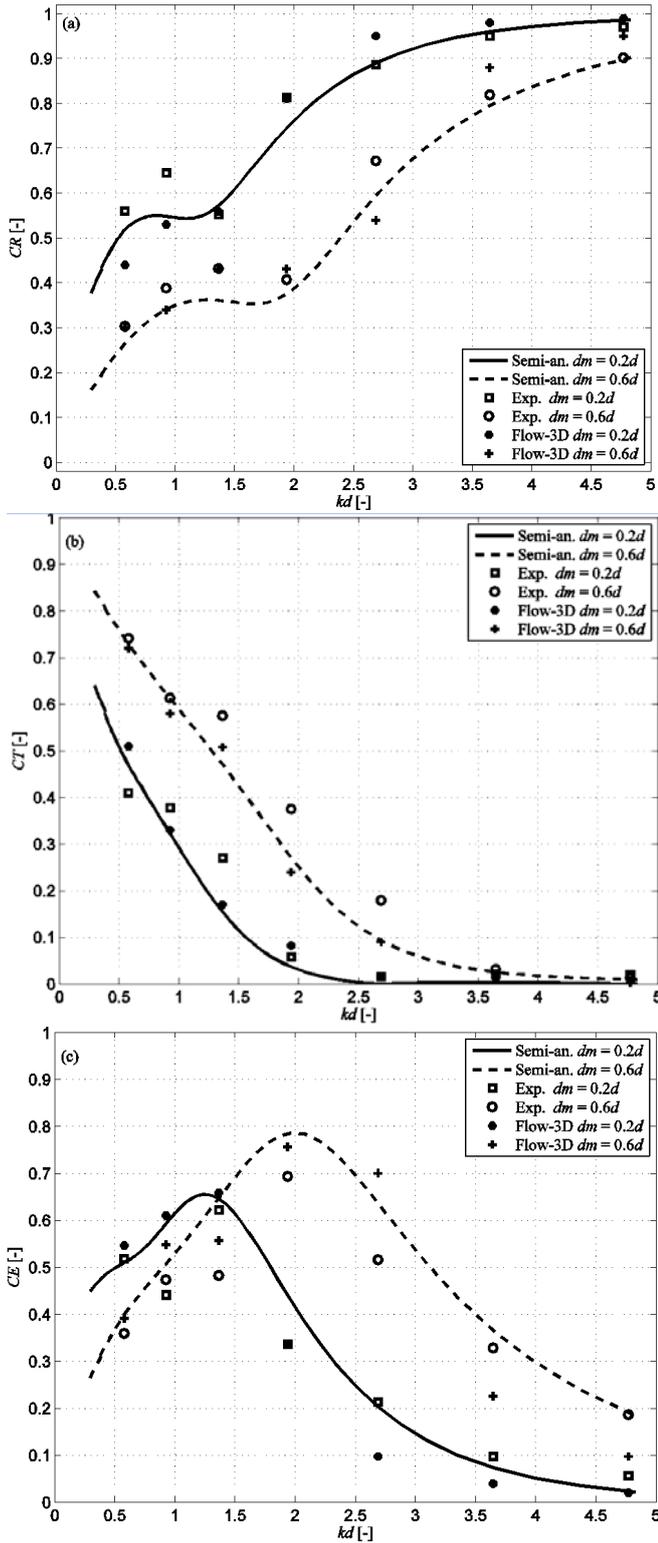


Fig. 2 Comparison of CFD (FLOW-3D) with experimental and semi-analytical results as function of ( $kd$ ) for  $\lambda/d = 0.25$  (a)  $CR$ , (b)  $CT$  and (c)  $CE$

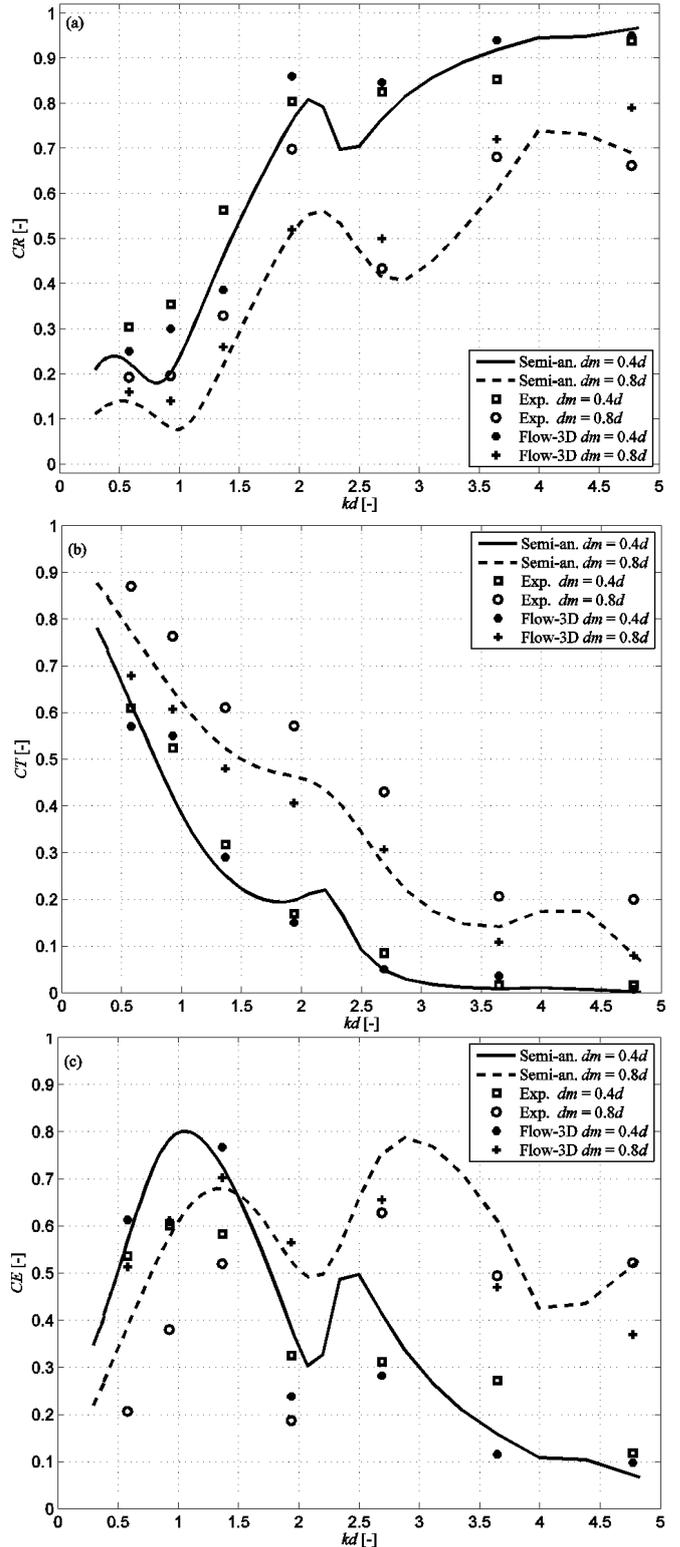


Fig. 3 Comparison of CFD (FLOW-3D) with experimental and semi-analytical results as function of ( $kd$ ) for  $\lambda/d = 0.75$ . (a)  $CR$ , (b)  $CT$  and (c)  $CE$

The numerical results for the reflection and transmission coefficients, show a good comparison (see Figs. 4 and 5) with the experimental and semi-analytical results which have been

reported by Ahmed [4]. It can be found that, the discrepancies of numerical results are in a range of less than +/- 10% for almost all of the experiments. A few points show discrepancies little more than +/- 10%, especially for cases presented in Fig 5. In general, there is a good agreement between the numerical and theoretical results, but it is also observed a small dispersion with experimental results.

The most notable differences between the numerical and experimental results occur for the transmission coefficient: the measured transmission coefficients are consistently higher than the numerical ones for lower wave numbers for both barrier spacing.

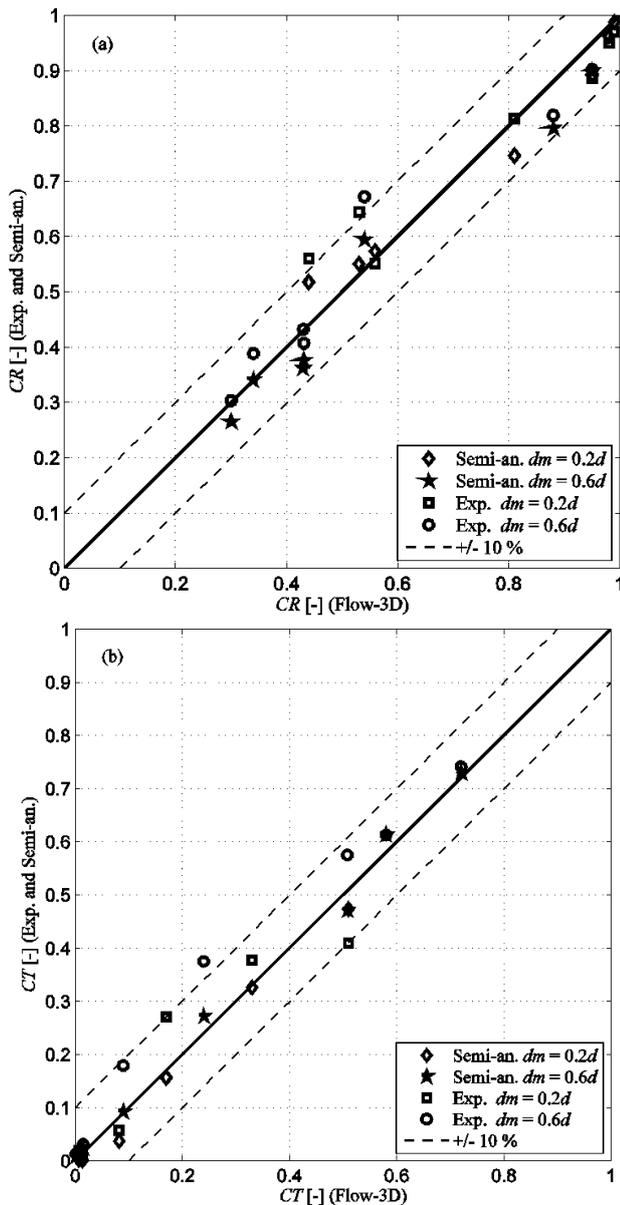


Fig. 4 Quality of CFD (FLOW-3D) vs. experimental and semi-analytical results for the same parameters and characteristic of Fig 2. (a) CR, (b) CT.

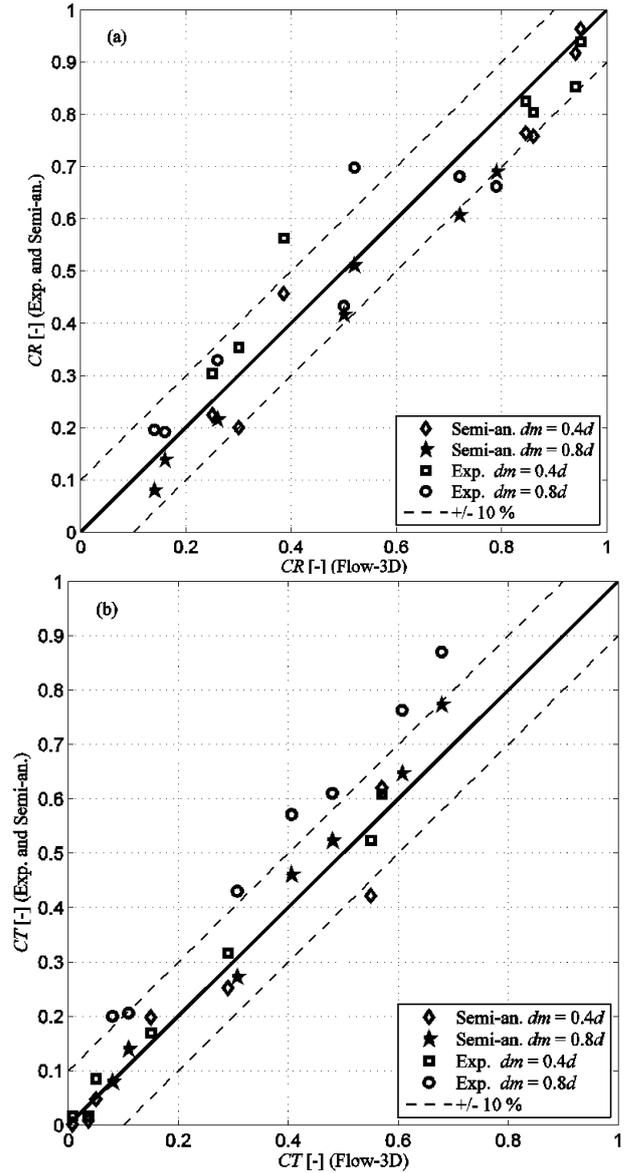


Fig. 5 Quality of CFD (FLOW-3D) vs. experimental and semi-analytical results for the same parameters and characteristic of Fig. 4 (a) CR, and (b) CT

### B. Influence of the Friction Factor 'f':

Hydrodynamic performance of a permeable barrier is generally affected by both the friction ' $f$ ' and the added mass ' $cm$ ' factor which have been discussed before in section II. But the added mass factor has only a minor influence and it can be taken as zero within this configuration [4].

An extensive variation of the friction factor ' $f$ ' over a full range of wave characteristics and geometry of slots is carried out. The friction factor ' $f$ ' has direct influence on the hydrodynamic performance of a permeable barrier as shown in Figs. 6-9.

It is clear that, the friction factor ' $f$ ' is dependent on the characteristics of waves and the contact surface area of the slots, but is not affected by spacing between the barriers. The values of the friction factor ' $f$ ' may differ for different wave characteristics and the contact surface area of the slots.

However, the friction factor ' $f$ ' has no influence on both  $CR$  and  $CT$  for short waves. It seems more than 5 for short waves, then decreases with decreasing the wave number and reached to about 1.5 and 2.5 for long waves when  $dm = 0.2d$  and  $0.6d$  respectively. The friction factor ' $f$ ' reasonably increases with increasing the opening area.

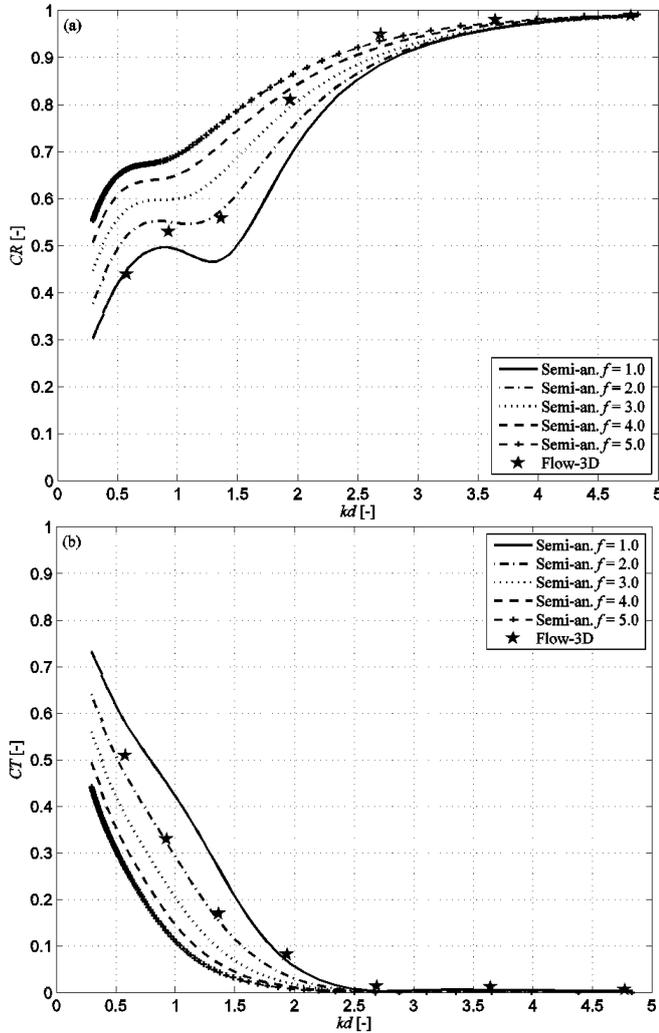


Fig. 6 Comparison of CFD (FLOW-3D) with semi-analytical results as function of  $(kd)$  for  $dm = 0.2d$ ,  $\varepsilon = 0.5$ ,  $\lambda/d = 0.25$ , various  $f$  and  $cm = 0$  (a)  $CR$  and (b)  $CT$

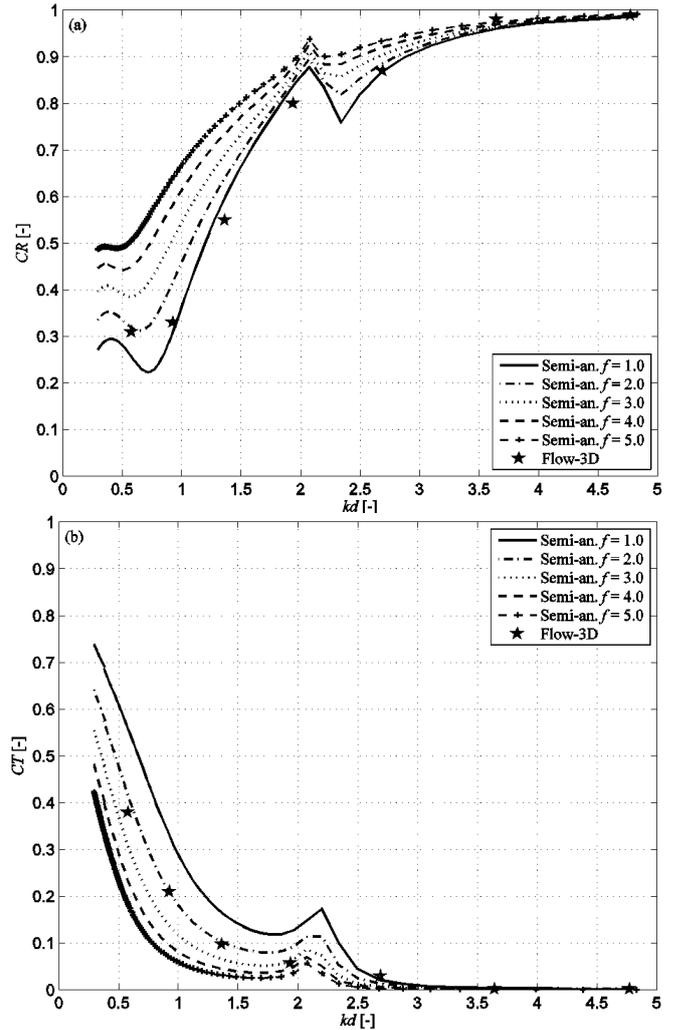


Fig. 7 Comparison of CFD (FLOW-3D) with semi-analytical results as function of  $(kd)$  for  $dm = 0.2d$ ,  $\varepsilon = 0.5$ ,  $\lambda/d = 0.75$ , various  $f$  and  $cm = 0$  (a)  $CR$  and (b)  $CT$

#### IV. CONCLUSION

Numerical simulations based on the Navier-Stokes equations have been used to simulate the wave interactions with a double vertical slotted wall barrier. The numerical model has been validated by comparisons with theoretical and experimental data of Ahmed [4]. The theoretical results based on the eigenfunction expansion method. The agreement is generally satisfying and indicates that the numerical model is capable to adequately reproduce most of the important features of the experimental and theoretical results under the same boundary conditions.

The discrepancy of numerical results of CFD (FLOW-3D) is in a range of +/- 10 % for most of experiments. A few points have a deviation of little more than +/- 10 %.

The friction factor ' $f$ ' is affected by the characteristics of waves and the geometry of the slots, but is not affected by the spacing between the barriers. The friction factor ' $f$ ' has no influence on both of  $CR$  and  $CT$  for short waves. Then, it

decreases with decreasing the wave number and increases with increasing the opening area.

Finally, the results of the numerical model (FLOW-3D) are acceptable for further investigations of the hydrodynamic performance of permeable barriers.

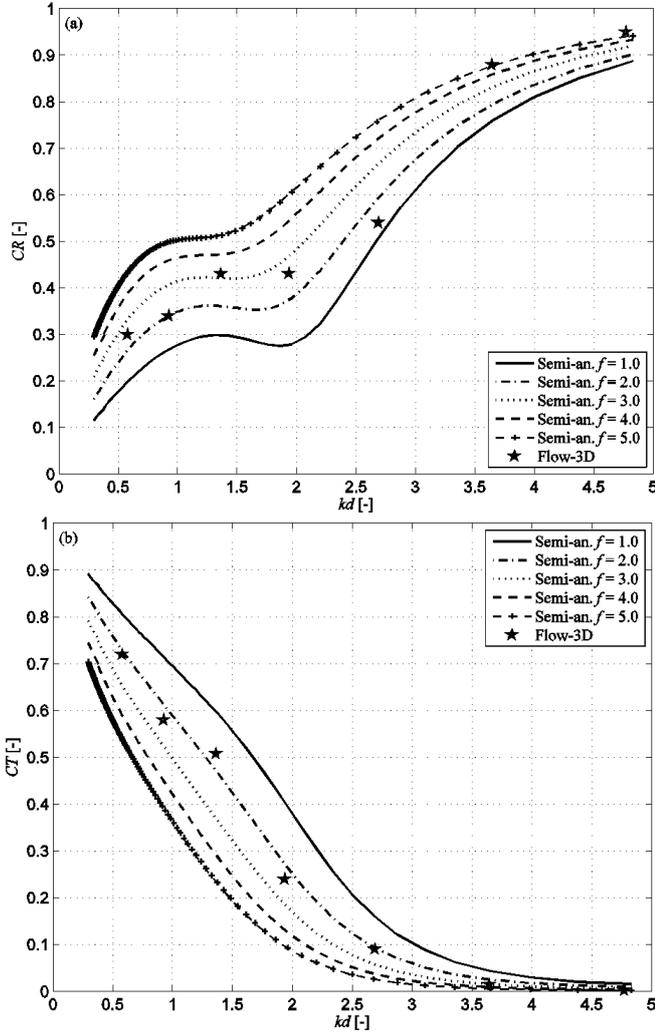


Fig. 8 Comparison of CFD (FLOW-3D) with semi-analytical results as function of  $(kd)$  for  $dm = 0.6d$ ,  $\varepsilon = 0.5$ ,  $\lambda/d = 0.25$ , various  $f$  and  $cm = 0$ . (a)  $CR$  and (b)  $CT$

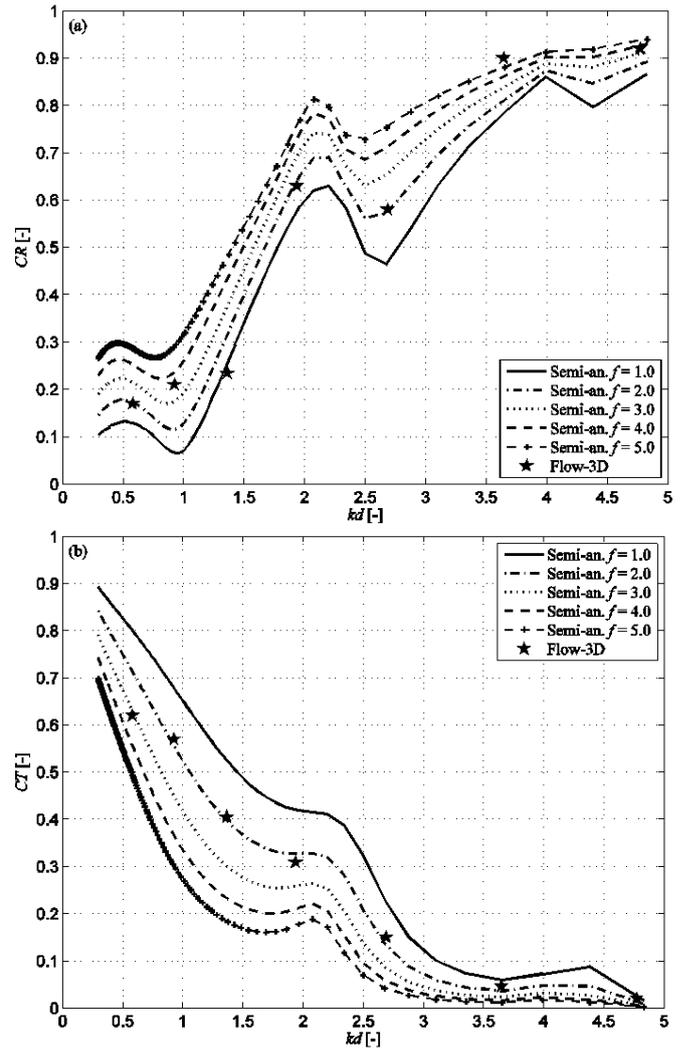


Fig. 9 Comparison of CFD (FLOW-3D) with semi-analytical results as function of  $(kd)$   $dm = 0.6d$ ,  $\varepsilon = 0.5$ ,  $\lambda/d = 0.75$ , various  $f$  and  $cm = 0$ . (a)  $CR$  and (b)  $CT$

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