

DISCHARGE COEFFICIENTS OF PIANO KEY WEIRS FROM EXPERIMENTAL AND NUMERICAL MODELS

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ABSTRACT

Piano Key Weirs (PKW) are relative new hydraulic structures of the labyrinth weir family. Their aim is to increase discharge capacities and hence to reduce upstream water levels in reservoirs or general flow systems. PKWs are in the focus of research investigations for some ten years and especially experimental models are used to determine discharge coefficients. During the last years also numerical 3D simulations are compared with those results from physical model tests. The present paper investigates results from a scaled experimental model and analyzes discharge coefficients of various PKW structures. The research focuses on the comparison of discharge coefficients for PKWs without and with geometrical adaptations at the downstream end at the top of the structure. Therefore, an investigation program is carried out to analyze the upstream free surface profiles and their influence on resulting discharge coefficients. Additionally, discharge coefficients from experimental models and numerical 3D simulations are compared to test the applicability of a chosen numerical VOF code. Therefore, the numerical model is an upscaled copy of the experimental model.

Keywords: Piano Key Weir, PKW, Discharge Coefficient, Experimental Model, Numerical VOF Simulation

1. INTRODUCTION

A Piano Key Weir (PKW) is a nonlinear weir type. A general design was developed by Blanc and Lempérière (2001) and Lempérière and Ouamane (2003). A PKW can be used to increase discharge capacity relative to linear weirs, and it is ideal for top-of-dam spillway control structure applications as the footprint space requirement is relatively small in comparison to the developed weir length geometry. A significant amount of research has been carried out during the last years using laboratory-scale physical models (Anderson 2011, Machiels *et al.* 2012). PKWs have been generally classified into four main geometric types (Type A, B, C, and D). A type A PKW features symmetrical keys relative to a transverse centerline axis. Type B features cantilevered apexes on the upstream end (outlet keys) and vertical apex walls on the downstream end (inlet keys). Type C is the opposite of the Type B with the cantilevered apexes on the downstream end. Type D is a rectangular labyrinth weir (vertical apex walls). Ribeiro *et al.* (2012) identified primary (crest length, head) and secondary (ratio of inlet and outlet key width and height, overhang length, height of parapet walls) parameters, having a significant effect on the PKW Type A discharge capacity. Figure 1 gives general PKW parameters. Discharge coefficients for free and submerged flow over PKWs were determined by Kabiri-Samani and Javaheri (2012). Dabling and Tullis (2012) evaluated and compared submerged head-discharge relationships for PKW Type A and labyrinth weirs. Machiels *et al.* (2011) analyzed flow characteristics over a PKW in a scaled experimental model. Anderson (2011) conducted a systematic study of PKW geometries and discharge efficiency. Anderson and Tullis (2012) evaluated PKW Type A head discharge behaviors for in-channel and reservoir-approach flow conditions. Oertel and Tullis (2014) compared experimentally determined discharge coefficients for several PKW types with those from a numerical 3D VOF model and found a good agreement and applicability of the VOF code.

Generally, discharge coefficients for a PKW can be determined by Poleni formula:

$$Q = \frac{2}{3} C_d L (2g)^{0.5} H_T^{1.5} \quad [1]$$

where Q = discharge, C_d = dimensionless discharge coefficient, L = total centerline crest length, g = acceleration due to gravity, H_T = total upstream energy head including velocity head $v^2/(2g)$.

2. EXPERIMENTAL AND NUMERICAL MODELS

2.1 Experimental model

The experimental PKW model was built up at Lübeck University of Applied Sciences' Hydraulic Laboratory within a tilting flume: total length $L_F = 9.90$ m, width $W_F = 0.80$ m, height $H_F = 0.80$ m. Upstream water levels were measured using three ultrasonic sensors (fabricate: General Acoustics, type: 635, accuracy: ± 1 mm). Figure 2 shows the experimental model setup. Discharges were determined by magnetic inductive discharge meter (MID, fabricate: Krohne, type: Optiflux, accuracy ± 0.1 l/s). Tested discharges started at $Q = 5.0$ l/s and were increased in 5 l/s steps up to $Q = 100.0$ l/s. The PKW models are made of acrylic material with a thickness of $T = 5.0$ mm. The total overfall length is $L = 4.667$ m, while the footprint length is $B_b = 0.2307$ m and the total weir length in flow direction is $B = 0.489$ m. Inflow and outflow chambers are $W_i = 0.105$ m and $W_o = 0.084$ m. Figure 3 and Table 1 give geometrical boundary conditions of the tested weir.

To identify the influence of geometrical variations of the weir, triangle (PKW2) and semi-circle (PKW3) expansions were fixed at the downstream crest end (see Fig. 3). The length of both tested expansions is 57.5 mm; the thickness is 5.0 mm.

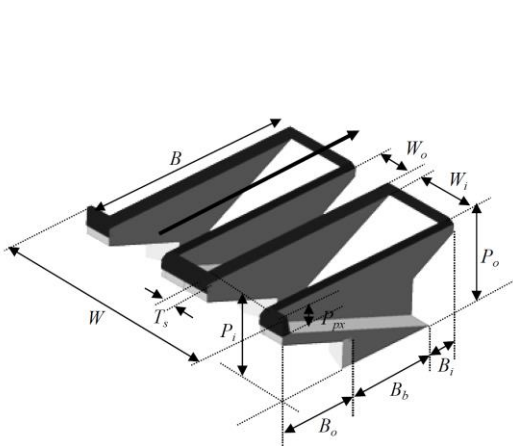


Figure 1. General PKW parameters (Pralong *et al.* 2011)

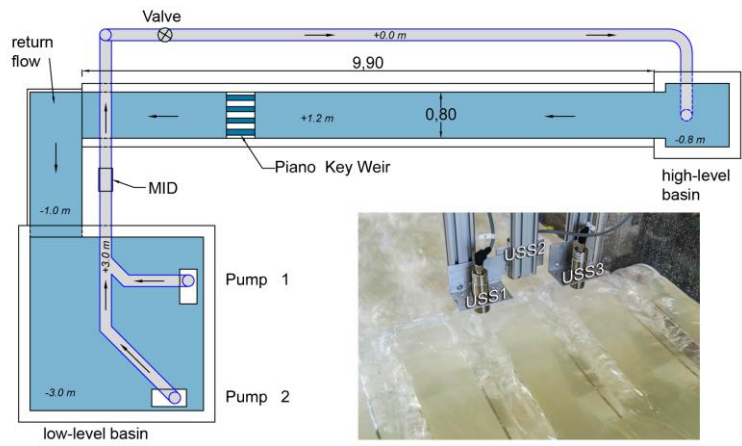


Figure 2. Schematic plan view on experimental water circulation system and ultrasonic sensors above PKW

Table 1. Summary of laboratory-scale PKW model dimensions

Test Weir dimensions [mm]			
P	196.9	L	4667.0
W	796.0	B_b	230.7
W_i	105.0	$B_o = B_i$	129.1
W_o	84.0	T_s	5.0

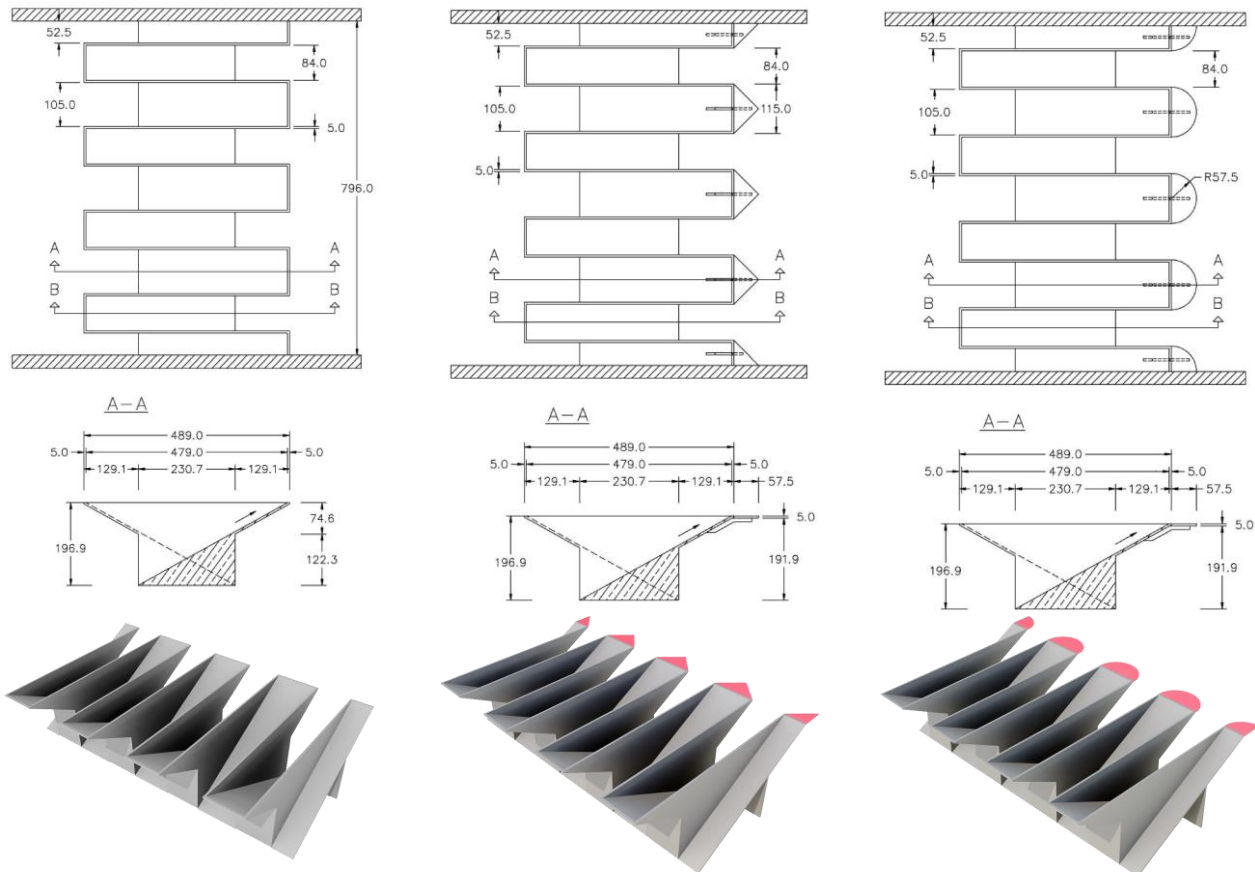


Figure 3. Sketches and dimensions of laboratory-scale PKW model (left: regular PKW1, middle: triangle expansion PKW2, right: semi-circle expansion PKW3), flow direction left to right

2.2 Numerical model

FLOW-3D, a commercial 3D CFD software, was used to simulate selective discharge conditions corresponding to the experimental model tests. FLOW-3D uses the Volume-of-Fluid (VOF) method to detect the flow's free surface (Hirt and Nichols 1981). To reduce calculation times laminar simulations were performed. An accurate applicability of FLOW-3D for free surface flows was confirmed in Bung *et al.* (2008), Oertel and Bung (2012), Oertel (2012), as well as Oertel and Tullis (2014). The flow domain of the numerical model was adapted to that of the experimental model, but up-scaled to prototype geometrical and hydraulic boundary conditions. As inflow boundary condition the discharge with a specified flow depth was given. At the downstream end of the model the outflow boundary condition was set. The model is made of one mesh block with a cell size of 0.05 m. In total approx. 2.25 Mio. cells represent the model area. The PK weir was included using a STL geometry file to guarantee an exact reproduction of the experimental model. Figure 3 illustrates the STL file and the mesh block for the numerical 3D model. Figure 4 gives more details and an example result of a numerical simulation.

2.3 Investigation program

To compare results from experimental and numerical models, a pre-defined investigation program was carried out. Within the scaled experimental model, a large number of discharges were investigated. Table 2 summarizes the investigation program. Within the numerical model selected discharges were investigated in prototype scale. To compare results for discharge coefficients it is not necessary to up- or downscale results from both models due to dimensionless units. For additional comparisons of selected flow depths, numerical model results were downscaled to those within the experimental model via Froude model. The investigation program was repeated for the three various investigated PKWs. Hence, 60 experimental model runs and 18 numerical model runs were carried out (78 in total).

Table 2. Investigation program, experimental and numerical model discharges for laboratory and prototype scale

Q_{exp} [l/s]	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
q_{exp} [l/(s×m)]	6.25	12.5	18.8	25.0	31.3	37.5	43.8	50.0	56.3	62.5	68.8	75.0	81.3	87.5	93.8	100	106.3	112.5	118.8	125.0
Q_{num} [m ³ /s]	-	-	4.74	6.32	-	9.49	-	-	-	15.8	-	-	-	22.1	-	-	-	28.5	-	-
q_{num} [m ³ /(s×m)]	-	-	0.59	0.79	-	1.19	-	-	-	1.98	-	-	-	2.77	-	-	-	3.56	-	-
$Q_{num,scaled}$ [l/s]	-	-	15	20	-	30	-	-	-	50	-	-	-	70	-	-	-	90	-	-
$q_{num,scaled}$ [l/(s×m)]	-	-	18.8	25.0	-	37.5	-	-	-	62.5	-	-	-	87.5	-	-	-	112.5	-	-

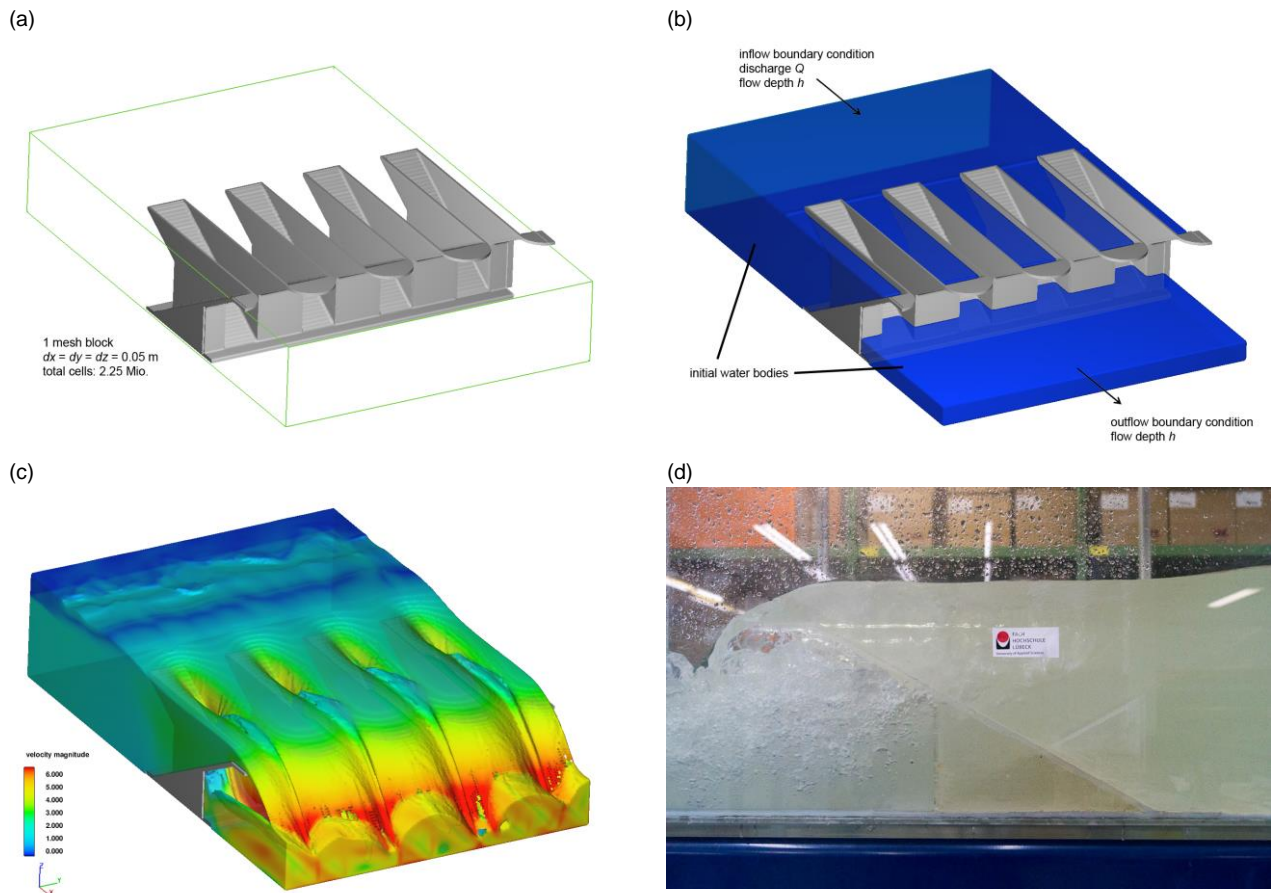


Figure 4. Example figures from numerical and experimental model for PKW3, (a) STL geometry file, (b) boundary conditions and initial water bodies, (c) flow velocities within num. model for $Q_{num} = 28.5 \text{ m}^3/\text{s}$ ($Q_{num,scaled} = 90 \text{ l/s}$), (d) water surface within exp. model for $Q_{exp} = 90 \text{ l/s}$

3. RESULTS AND DISCUSSION

3.1 Discharge coefficients

For the experimental model, discharge coefficients were analyzed by measuring water surface profiles upstream the PKW via ultrasonic sensors. These water levels were used to calculate relative overfall heights for the Poleni formula $Q = 2/3 C_d L (2g)^{0.5} H_T^{1.5}$, while L is the total weir length at the crest of the PKW. Figure 5 gives the discharge coefficients for the experimental model in comparison with Anderson and Tullis (2012), as well as discharge coefficients for the numerical model.

It can be shown, that for investigated discharge $H_T/P > 0.15$ results from the experimental as well as the numerical model are close to those collected by Anderson and Tullis (2012); but they cannot be reproduced exactly. It must be noticed, that

the investigated weir of the present study differs slightly from those found in the literature (e. g. material thickness). Hence, discharge coefficients will differ. For $H_T/P > 0.15$ a very good reproduction of results collected within the experimental model can be found for numerical simulation results.

For $H_T/P < 0.15$ a significant deviation can be found for discharge coefficients from the experimental model. These values are found to be 10 to 20 % larger than coefficients from the numerical model or Anderson and Tullis (2012). It can be assumed that these deviations are a result of data analysis as well as measurement technique. In the present study it can be shown that small changes of the position of the analyzed energy heads and the inclusion of velocity heads will influence the results significantly. Figure 6 gives water surface levels for all numerically investigated discharges and only small deviations can be found. Hence, again it can be assumed, that varying discharge coefficients result from very small differences in flow depth and velocity. To explain this effect in detail and to clarify how to choose the exact position upstream the PKW for data analysis, further investigations are necessary. Also the numerical model should be modified to clarify the inflow characteristics at the channel's upstream boundary. It can be assumed, that the longitudinal model size is too small for generating a fully developed velocity profile. Hence, further studies are also necessary within the numerical model.

Comparing discharge coefficients of varying PKW types (without and with expansion, PKW1, PKW2, PKW3) it can be noticed that the influence of the expansion's shape (triangle or semi-circle) can be neglected (see Fig. 5). But generally it can be found, that expansions will decrease the efficiency of the PKW weir for small discharge events ($H_T/P < 0.15$), compared with a regular shape (PKW1) without expansions. Hence, discharge coefficients are smaller for PKW2 and PKW3 for small discharges, where the weir will be overtopped at all Piano Key segments. For increasing discharges ($H_T/P > 0.15$) discharge coefficients for all three investigated types equals and follow a comparable trend between $0.15 < H_T/P < 0.40$.

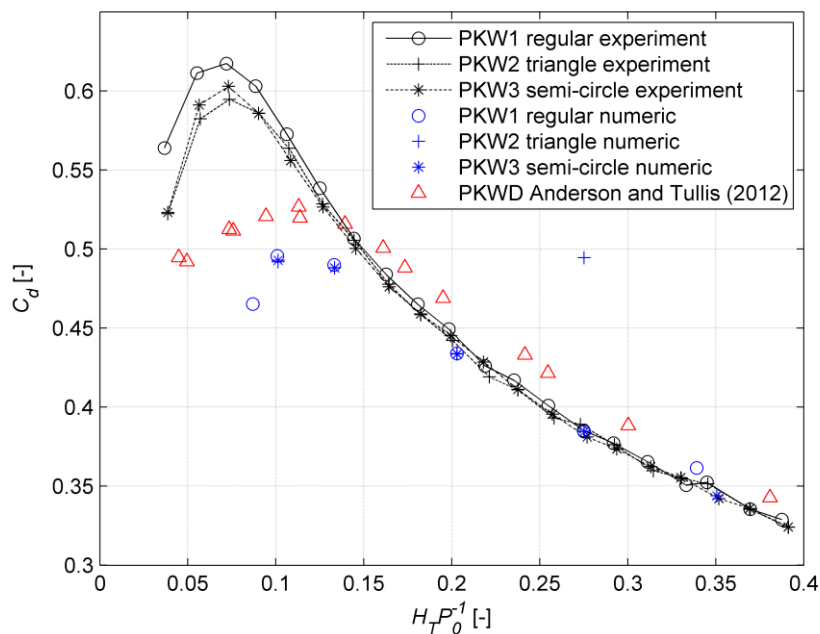


Figure 5. Discharge coefficients from experimental and numerical models

3.2 Selected flow depths

For all numerically investigated PKW model runs water surface elevations will be compared with those collected within the experimental model. Figure 6 gives the comparison of all results. It can be shown, that the numerical CFD model with included VOF method is able, to reproduce experimental model results accurately. Hence, it must be clarified in further studies how the position upstream the PKW, from which discharge coefficients will be calculated, influence the investigation results. Also a precise analysis of velocity head consideration is necessary to find exact discharge coefficients. Generally, it can be noticed, that FLOW-3D can be used as fast and easy-to-use numerical analysis tool.

Within further studies detailed processes, like e. g. detailed turbulent flow structures at the upstream PKW inflow area can be analyzed by including turbulence models or by running LES. The experimental model results show, that time-dependent fluctuations can be observed, which might influence discharge coefficients and the flow situation in general. A LES simulation maybe can reproduce these unsteady flow fluctuations and give additional information.

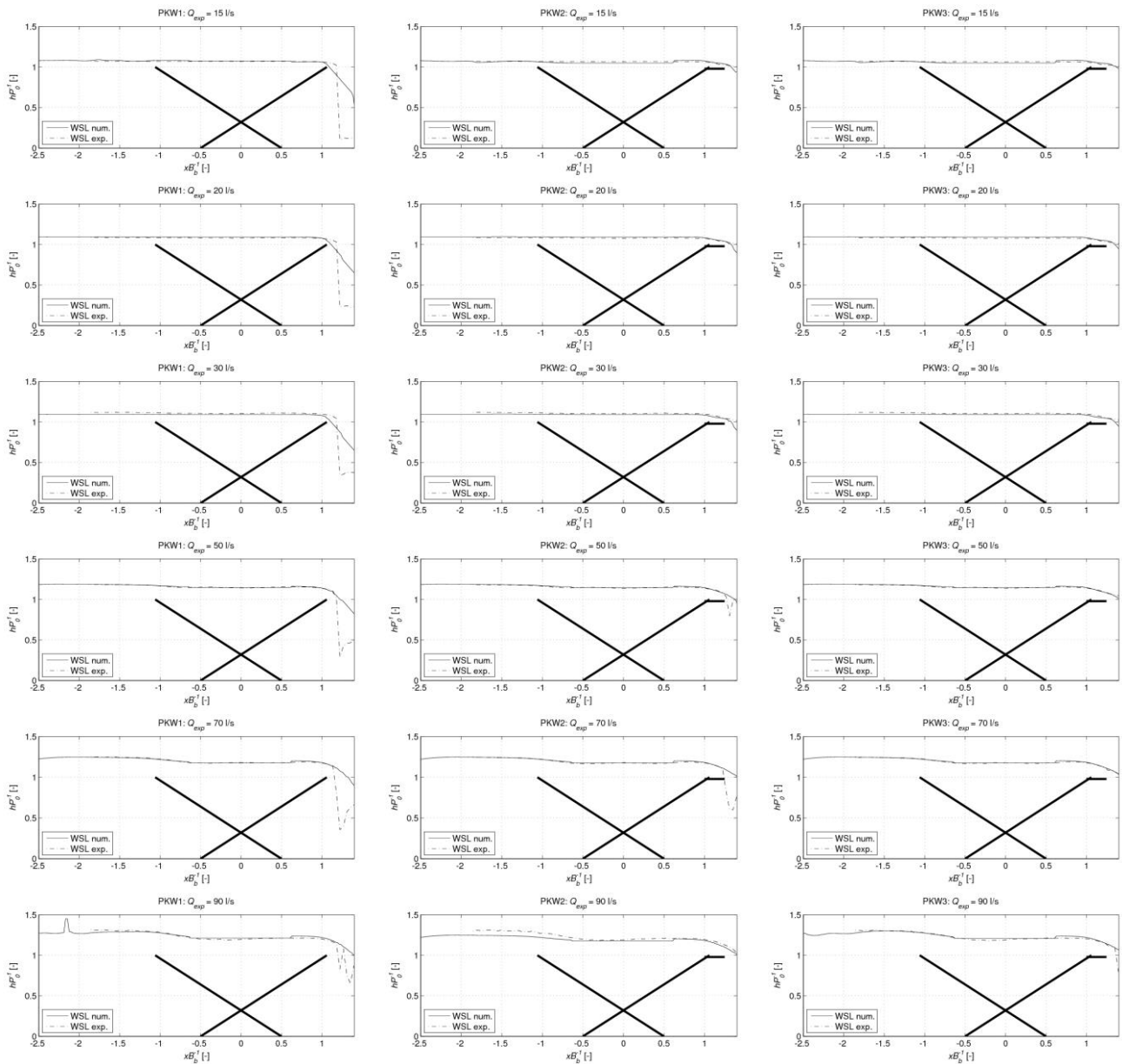


Figure 6. Comparison of WSL from experimental and numerical model, left: regular PKW1, middle: triangle expansion PKW2, right: semi-circle expansion PKW3, various discharges, scaled to experimental model size, longitudinal section in flume's centerline

4. SUMMARY AND CONCLUSION

The present study investigates discharge coefficients and water surface levels for various PKW types. Therefore, an experimental model was built to measure flow depths for varying discharges up to 100 l/s. Additional numerical 3D CFD simulations were made to reproduce water surface profiles and discharge coefficients. Collected data were compared with data from the literature.

It can be found, that the numerical CFD model is able to reproduce flow depths of the experimental model accurately. But when calculating discharge coefficients, only for larger discharge events ($H_t/P > 0.15$) a good reproduction can be noticed. For $H_t/P < 0.15$ a significant deviation of the experimental model results can be found. Discharge coefficients are 10-20 % larger for the experimental model compared with those from the numerical model or the literature. It could be shown that small differences in water surface levels have a significant influence on the calculated discharge coefficients. Hence, a detailed study on data collection is necessary, to identify the exact measuring position upstream the PKW.

REFERENCES

- Anderson, R. M. (2011). Piano Key Weir Head Discharge Relationships. *All Graduate Theses and Dissertations*, Paper 880, Utah State University.
- Anderson, R. M.; Tullis, B. P. (2012). Piano Key Weir: Reservoir versus Channel Application. *Journal of Irrigation and Drainage Engineering*, 138, 773-776.
- Blanc, P.; Lempérière, F. (2001). Labyrinth spillways have a promising future. *Hydropower & Dams*, 8(4), 129-131.
- Bung, D.B.; Hildebrandt, A.; Oertel, M.; Schlenkhoff, A.; Schlurmann, T. (2008). Bore propagation over a submerged horizontal plate by physical and numerical simulation. *Proc. 31st Int. Conference on Coastal Engineering*, Hamburg, Germany, 3542-3553.
- Dabling, M. R.; Tullis, B. P. (2012). Piano Key Weir Submergence in Channel Applications. *Journal of Hydraulic Engineering*, 138(7), 661-666.
- Hirt, C. W.; Nichols, B. D. (1981). Volume of Fluid (VOF) method for the dynamics of free boundaries. *J. of Computat. Phys.*, 39(1), 201-255.
- Kabiri-Samani, A.; Javaheri, A. (2012). Discharge coefficients for free and submerged flow over Piano Key weirs. *Journal of Hydraulic Research*, 50(1), 114-120.
- Lempérière, F.; Ouamane, A. (2003). The piano key weir: a new cost-effective solution for spillways. *Hydropower & Dams*, 10(5), 144-149.
- Machiels, O.; Ercicum, S.; Dewals, B.; Archambeau, P.; Piroton, M. (2011). Experimental observation of flow characteristics over a Piano Key Weir. *Journal of Hydraulic Research*, 49(3), 359-366.
- Oertel, M.; Bung, D. B. (2012). Initial Stage of two-dimensional dam-break waves: laboratory vs. VOF. *Journal of Hydraulic Research*, 50(1), 89–97.
- Oertel, M. (2012). Crossbar block ramps – Flow regimes, flow resistance, energy dissipation, stability. *Habilitation thesis*, University of Wuppertal, Germany.
- Oertel, M.; Tullis, B. P. (2014). Comparison of Piano Key Weir Discharge Coefficients from experimental and numerical models. *Proc. 3rd IAHR European Congress*, Porto, Portugal.
- Pralong, J.; Vermeulen, J.; Blancher, B.; Laugier, F.; Ercicum, S.; Machiels, O.; Piroton, M.; Boillat, J. L.; Leite Ribeiro, M.; Schleiss, A. J. (2011). A naming convention for the Piano Key weirs geometrical parameters. *Proc. Intl. Conf. Labyrinth and Piano Key Weirs*, Liege, Belgium.
- Ribeiro, M. L.; Pfister, M.; Schleiss, A.; Boillat, J.-L. (2012). Hydraulic design of A-type Piano Key Weirs. *Journal of Hydraulic Research*, 50(4), 400-408.
- USDI (1980). Hydraulic Laboratory Techniques. *A Water Resources Technical Publication*, U.S. Department of the Interior, Water and Power Resources Service, Denver, Colorado.