



## Continuous or catastrophic solid-liquid transition in jammed systems

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**Abstract:** Pasty materials are of general interest in industry (gels, creams, foams, drilling fluids, concrete, paints, foodstuffs, etc) and in earth science (mud or debris flows, lahars, soil liquefaction, etc). These materials are intermediate between solids and liquids either in terms of their internal structure (disordered but jammed) or from a mechanical point of view. Our results indicate that the type of behavior of a particulate system (soils, suspensions, clays, etc) depends on the relative importance of the energy supplied to it and its “jamming state” which evolves in time. For example a soil can transform into a liquid if sufficient energy (shaking, shearing, etc) is supplied to it. This suggests that a bridge exists between as yet separated disciplines such as soil mechanics and fluid mechanics of suspensions, which will be useful for modelling catastrophic solid-liquid transitions occurring in particulate systems.

In nature and industry there exist a lot of materials of behavior intermediate between solids and liquids: muds, lavas, snow, fresh concrete, paints, mayonnaise, foams, emulsions, melt chocolate, gels, varnishes, sewage sludges, drilling fluids, etc. Their mechanical properties are ubiquitous, and they can appear as solid or liquid depending on circumstances. The transition from one state to the other is of critical importance in various cases, in particular when it occurs abruptly. For example self-compacting concrete, paints or drilling fluids flow like low viscosity liquid, rapidly “gelify” at rest thus keeping coarse particles in suspension, but can liquefy again if submitted to some vibration so that particles sedimentate and lead to material heterogeneities [1]. Also subaerial or submarine landslides can turn to mudflows or debris flows as they move downwards [2-4], sensitive clayey soils can suddenly liquefy and flow over large distances [5-6], or mountain streams incorporating debris or ashes can turn to devastating mud or debris flows or lahars [7-8]. Here we show experimentally that these

different states and evolutions can be obtained with the same material depending on boundary conditions, restructuration time or solid content. These trends are reproduced by numerical simulations using a model simply taking into account time changes of material viscosity. This provides a general frame for describing the catastrophic solid-liquid transition of industrial pastes or natural soils, and suggests a continuity between fluid mechanics and soil mechanics based on the evolution of the ratio of the energy supplied to the material and its “jamming energy”.

In mechanics the above materials are generally considered as yield stress fluids, i.e. able to remain indefinitely at rest (in contrast with simple liquids) if the force applied to them is insufficient but capable to flow under a larger force [9]. Along with granular flows, these materials arouse the interest of physicists since they could constitute a fourth state of matter (besides gas, liquids and solids), i.e. the jammed systems [10]. This jammed state results from the fact that the elements (particles, bubbles, droplets, etc) confined in a certain volume form a continuous network of interactions throughout the sample, which must be broken for flow to occur. The usual, basic description of their mechanical behavior involves a simple, continuous transition from a solid to a liquid behavior beyond the yield stress [9]. However it was shown recently that the solid-liquid transition in pasty and granular materials occurs in the form of a viscosity bifurcation [11-12]: under a shear stress ( $\tau$ ) smaller than a critical stress ( $\tau_c$ ) the fluid evolves more or less rapidly towards complete stoppage (infinite viscosity), while under a slightly larger stress the fluid evolves towards rapid flow with a low viscosity. This implies there is a discontinuity in the steady state viscosity at the solid-liquid transition. In this context  $\tau_c$  is an apparent yield stress which increases with the time of rest before flow. This means that the behavior of these intermediate materials is basically time-dependent, so that the yielding character is intimately linked to thixotropic effects : i.e., a viscosity increase at rest, and a viscosity decrease under flow. These effects have for long been identified with drilling fluids, paints or cement pastes, but these results show that they must be taken into account even for dealing with the solid-liquid transition in materials for which thixotropy does not appear significant. While there have been some studies focusing on quasi-steady flows (e.g. constant shear), very little studies investigated a highly unsteady situation with strong interactions between thixotropy and flow. Present study focuses on well-defined highly unsteady flow conditions, with a broad range of industrial and geophysical applications such as landslides, mudflows, concrete placement, etc.

In this frame it is valuable to focus on a typical yield stress fluid with marked thixotropic character, for the physical effects to be enhanced and more clearly identified. In this aim we used suspensions of bentonite (see [13]), a natural clay, at different solid mass concentrations ( $\phi$ ). To reproduce the generic practical situation in which a force is suddenly applied to a mass of material, we poured a given volume of suspension in a dam reservoir at the top of an inclined channel (width: 34 cm, slope: 15°, rough surface to avoid slip). This volume of fluid was left at rest for some time ( $T$ ) and then we abruptly lifted up the (vertical) gate (Figure 1). This induced a slight (vertical) shearing of the material which likely did not affect its downstream flow. Sedimentation did not occur within the duration of our tests with such materials. We carried out a series of systematic experiments under different times of rest ( $T$ ), during which the fluid restructured, and for various solid fractions ( $\phi$ ). Here we only present the main qualitative trends of the resulting flows which are described in further details in a research report [14]. Four flow types were observed (Figure 2):

\* Regime I: For small  $\phi$  or short  $T$  the fluid was submitted to a rapid acceleration at the gate opening and flowed rapidly downstream; the flow aspect was that of a gravity current of a simple liquid [15] (Fig. 3a).

\* Regime II: For intermediate  $\phi$  or  $T$  the fluid initially flowed rapidly then its velocity abruptly decreased after some distance; the layer of material slowly flowed during some time then completely stopped; the flow aspect was typical of a layer of a yield stress fluid released over an inclined plane and which stopped flowing when the wall shear stress balanced the yield stress [16-17] (Fig.3b)

\* Regime III: For large  $\phi$  or  $T$  the whole material started to flow, then separated into a tail which remained attached to the initial dam and stopped flowing, and a front which went on flowing downstream over some distance then stopped (Fig.3c); in the front part the shear was localized along the wall, the rest of material above more or less moving in mass (this was not a wall slip since the surge front part left behind it a thin layer of material); these characteristics were reminiscent of the aspect of some landslides; in some cases the moving part advanced over a larger distance than in the Regime II, for example reaching the downstream extremity of the channel with some velocity.

\* Regime IV: In that case the material slightly deformed but never flowed downstream even after several days (Fig.3d).

These flow regimes surprisingly correspond to the overall motion characteristics usually observed with material types *a priori* considered as distinct: (I) sudden release of a simple liquid, (II) flow and stoppage of a simple pasty (yield stress) fluid, (III) landslide, (IV)

deformable solid. Our results show that, in contrast with the usual separation of materials in distinct categories, the apparent behavior of a jammed material under continuously varying conditions can cover all possible liquid or solid aspects.

In order to have a clearer view of the physical origin of these effects a possibility consists in attempting to reproduce them with a physically-meaningful model. In this aim we can use a simple model [11] proposed for explaining viscosity bifurcation effects, and which relies on classical thixotropy concepts [18]: the instantaneous viscosity ( $\mu = \tau/\dot{\gamma}$ , in which  $\dot{\gamma}$  is the rate of shear) of the fluid is a function of the actual *state of structure*  $\lambda$ , which expresses as  $\mu = \mu_0(1 + \lambda^n)$ , in which  $\mu_0$  and  $n$  ( $>1$ ) are two materials parameters. The thixotropic character of the fluid has its origin in the time variations of the state of structure, which result from the competition between restructuration process at a rate mainly depending on material properties and destructuration process at a rate proportional to the flow rate ( $\dot{\gamma}$ ):

$$\frac{d\lambda}{dt} = \frac{1}{\theta} - \alpha\lambda\dot{\gamma} \quad (1)$$

in which  $\theta$  is the characteristic time of restructuration. It may be shown [19] that under controlled stress such a material exhibits an apparent yield stress ( $\tau_c$ ) which increases with the structure state at the initial instant denoted  $\lambda_0$ :

$$\tau_c(\lambda_0) \approx \frac{\mu_0}{\alpha\theta} \lambda_0^{n-1} \quad (\text{for } \lambda_0 \gg 1) \quad (2)$$

Moreover, under controlled shear stress, stable can only be obtained for a shear rate larger than the critical value  $\dot{\gamma}_c = (n-1)^{1/n}/\alpha\theta$ . We deduce that  $\alpha\theta$  is an important characteristic time of the model, which in particular decreases as the solid fraction increases (since  $\dot{\gamma}_c$  increases [20]). Previous thixotropy models had been mainly compared with few macroscopic data but it was shown with the help of coupled Magnetic Resonance Imaging-rheometry that this model not only predicts the peculiar qualitative trends of paste flows (viscosity bifurcation, shear localization) but is also capable to well reproduce local flow characteristics under steady and transient conditions [19]. Equation (2) gives a first, though incomplete, explanation to the transition from regime III to regime IV: when the time of rest (or equivalently  $\lambda_0$ ) or the concentration are too large (or equivalently when  $\alpha\theta$  is too small) the apparent yield stress is so large that the mud does not start to flow.

We implemented this constitutive equation in the software *Flow 3D* and simulated these flows (in 2D, i.e. without lateral walls) under different values of  $\alpha\theta$  and initial values of the restructuration ( $\lambda_0$ ). The computational results are qualitatively similar to the experimental ones. In particular, exactly the same flow regimes can be identified (Figure 4) with decreasing  $\alpha\theta$  (and thus increasing minimum apparent yield stress, which also increases with the solid fraction) and increasing  $\lambda_0$ . Thus a phase diagram similar to that of Fig.2 could be drawn in a plane  $\lambda_0$  vs  $\alpha\theta$ . Using now the same software applied to a 3D flow motion, hence taking into account the lateral walls, the results appeared to be even more similar to our experimental results. The qualitative agreement between our simulations and reality means that the basic physical ingredients of this model are likely to be at the origin of the observed phenomena. This suggests that, in contrast with usual models devoted to a specific field, a model of this type is capable to reproduce the different stages of flow of natural mass movements which turn from solid to liquid or from liquid to solid.

We computed the total kinetic energy of the system in time for the different flow regimes with the help of the software (cf. Figure 5). The different regimes observed also correspond to strictly different time evolutions of the kinetic energy of the system, which provides a practical means for distinguishing them. A striking point is the peculiar evolution of the kinetic energy in the regime III: after a long period during which it does not vary, the kinetic energy dramatically increases and becomes much larger than the corresponding kinetic energy in the regime II under the same boundary conditions but with lower initial apparent viscosity. These trends are in perfect qualitative agreement with our experimental observations, which further confirms the ability of our model to describe various complex phenomena and in particular the above unexpected effect (regime III) which precisely corresponds to catastrophic events observed in practice. Note that the flow characteristics in each regime completely differ: in regime II the fluid is more or less homogeneously sheared; in regime III, when the kinetic energy is large, the fluid is mainly sheared in a thin layer close to the solid plane which ensures the rapid motion of a rigid mass of material above it.

More generally our results suggest that there is a continuity of a liquid suspension behavior towards the soil behavior as the solid fraction or the restructuration time (i.e. the *degree of jamming*) increases. The particle rearrangements during restructuration at rest leads to the formation of a stronger network of interactions somewhat analogous to the strengthening effect resulting from the increase of interaction number by unit volume when increasing the

solid fraction. In fact we can remark that existing MRI data concerning the internal flow characteristics of colloidal suspensions in steady state simple shear lead to an analogous conclusion. The shear localizes in a region of thickness decreasing with the solid fraction or with the restructuration time while the shear rate remains almost uniform in the sheared region [20-21]. This implies that, under a given rotation velocity of the inner cylinder, the material will appear (cf. Fig. 6) as a liquid for a low degree of jamming, as a yield stress fluid for an intermediate value, and plastic (with a shear localized along the larger stress region) for a large degree of jamming. However a material with a large degree of jamming may now appear as a yield stress fluid or as a liquid under a sufficiently large rotation velocity. Conversely a material with a low degree of jamming may appear as plastic solid under a sufficiently low rotation velocity.

In summary the present work suggests that the flow regime mainly depends on the relative importance of the energy supplied to the system and the initial degree of jamming of the material, which may be related to the average depth of potential wells of interaction between the particles of the suspension.

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## Captions

Figure 1: Initial conditions : the material at rest over the inclined plane and behind the transparent gate (dam) which will be lifted up very rapidly at the initial instant.

Figure 2: Qualitative distribution of the different flow regimes observed in our experiments as a function of the restructuring time and the solid fraction of the material (for a constant mass of fluid (3.7kg)).

Figure 3: Typical aspects of the flow in the different regimes: (a) Type I, simple liquid wave ( $\phi = 4.1\%$ ;  $T = 5\text{min.}$ ); (b) Type II, yield stress fluid layer ( $\phi = 16.4\%$ ;  $T = 1\text{min.}$ ); (c) Type III, landslide ( $\phi = 6.4\%$ ;  $T = 40\text{min.}$ ); (d) Type IV, solid at rest ( $\phi = 16.4\%$ ;  $T = 1035\text{min.}$ ).

Figure 4: Aspect of the flow from 2D simulations with Flow 3D of the channelized flow after a certain time, as described in the text, under rheological conditions:  $n = 1.067$  (the value deduced from complete measurements in [19]),  $\mu_0 = 0.1\text{Pa.s}$  and (a)  $\lambda_0 = 100$ ,  $\alpha\theta = 0.1$  (Regime I); (b)  $\lambda_0 = 10000$ ,  $\alpha\theta = 0.05$  (Regime II); (c)  $\lambda_0 = 1000000$ ,  $\alpha\theta = 0.025$  (Regime III); (d)  $\lambda_0 = 100000000$ ,  $\alpha\theta = 0.0125$  (Regime IV). The colors correspond to the different values of the structure parameter.

Figure 5: Total kinetic energy of the system as a function of time in the 2D simulations described in Fig.4 and corresponding to the different flow regimes described in the text.

Figure 6: Sketches of the velocity profile in a suspension in a Couette flow under a given rotation velocity of the inner cylinder for increasing degree of jamming (from left to right).

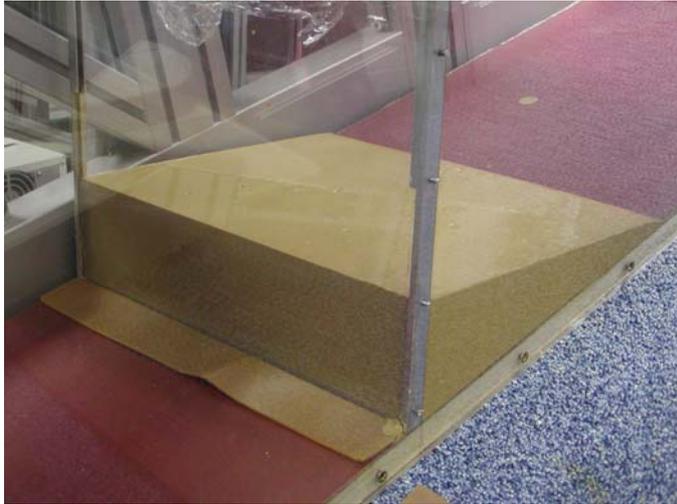


Figure 1

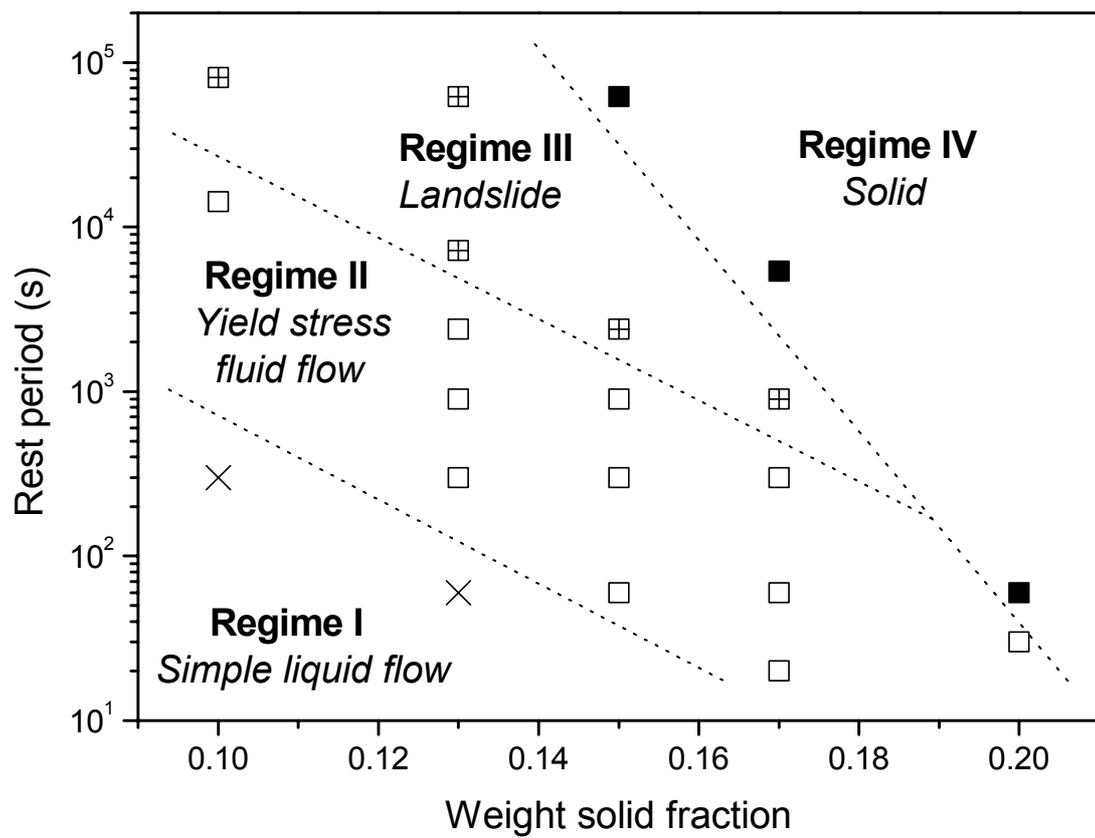
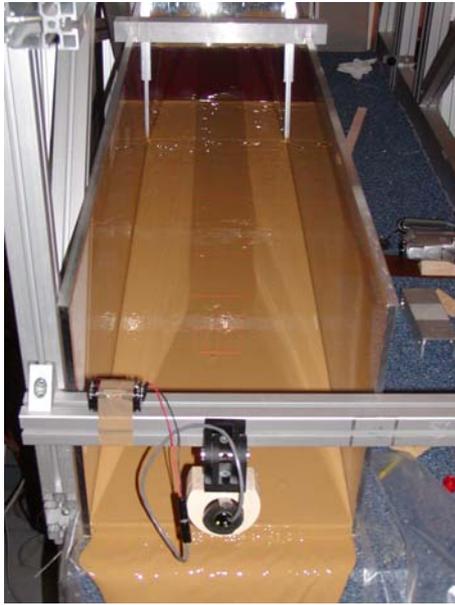


Figure 2



(a)



(b)



(c)



(d)

Figure 3



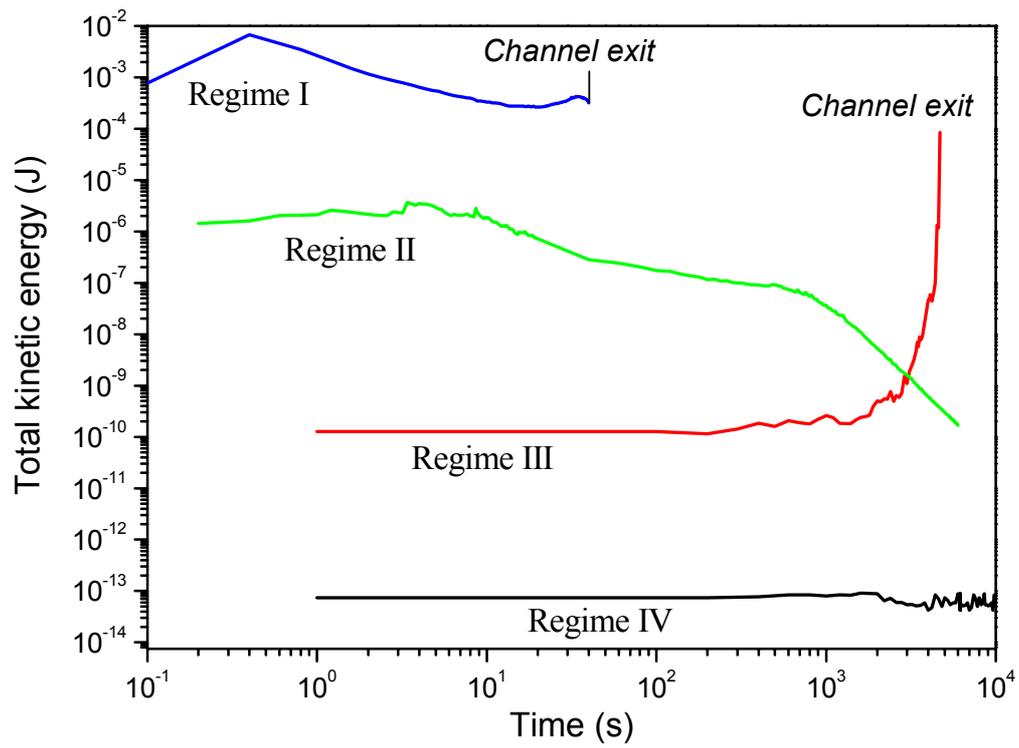


Figure 5

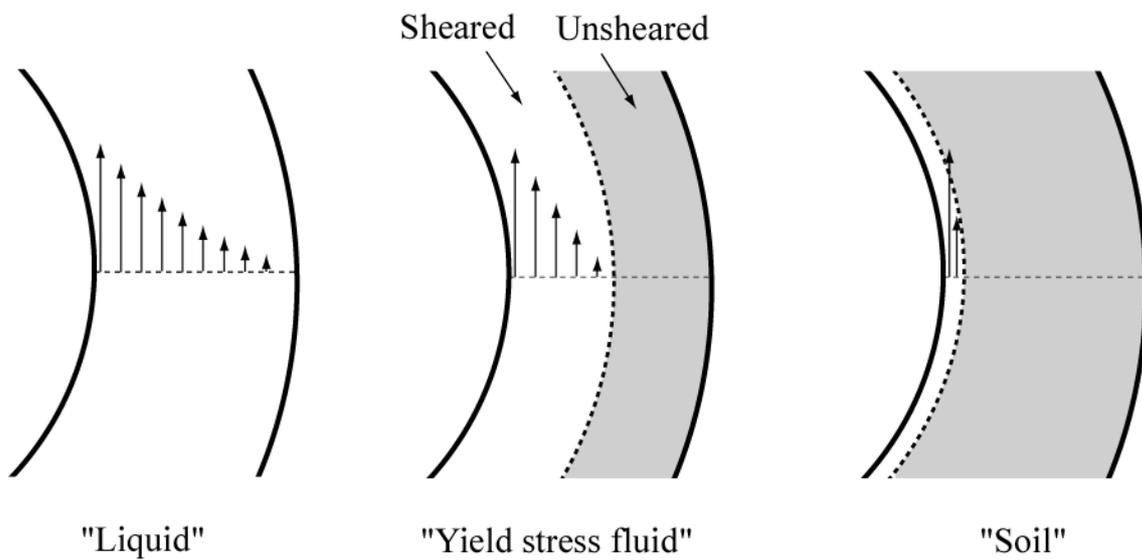


Figure 6