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Internal Wave Generation in Flow3D Model

Kwang-Oh Ko[1]
[1] Hyundai Engineering & Construction Company, R&D Center,
Yongin-Si, Gyounggi-Do, Korea

Jun-Woo Choi[2]
[2] Korean Institute of Construction Technology, Dept. of Water Resource & Environment,
Ilsanseo-Gu, Gyounggi-Do, Korea

Sung-Bum Yoon[3]
[3] Hanyang University, Dept. of Civil & Environ. Eng.,
Ansan-Si, Gyounggi-Do, Korea

Chang-Beom Park[4]
[4] Hyundai Engineering & Construction Company, R&D Center,
Yongin-Si, Gyounggi-Do, Korea

ABSTRACT

For the internal wave generation in the Navier-Stokes equation model using the VOF scheme, Lin and Liu (1999) developed an internal wave-maker method for which a mass source function of the continuity equation was used to generate target wave trains. Also, Choi and Yoon (2009) developed the internal wave generation scheme using momentum source to produce the target waves, which is applied to the RANS equation model in a CFD code, FLUENT.

In this study, two approaches including mass source and mass flux methods were devised for the application in a CFD code, FLOW-3D. Also, the sponge layer scheme using the porous media was introduced to remove the reflected waves into the computational area as proposed by Choi and Yoon (2009). The achieved results were compared to the analytical solutions of the Airy theory, Stokes' 2nd order and Stokes' 5th order theory. Also, the self-adaptive wave generator was devised to produce the accurate mass flux according the changes of the surface elevation in which the vertical profile of horizontal particle velocities are changed based on the position of free surface. This method will be expanded to the application of short waves, which the vertical particle velocities are more important for the accurate generation of free surface than that in case of long wave. The comparisons between the numerical results and the analytical solutions showed the good agreements not only the free surface elevation but the vertical profile of horizontal particle velocities. This achievement can be applied to the 3 dimensional wave simulations such as the wave tranquility study in the harbor area and the wave propagation from the offshore to the near-shore.

KEY WORDS: Internal wave generation; sponge layer; Flow 3D model; mass source, mass flux

INTRODUCTION

Numerical simulations of wave deformation have not been applied because the computational capacity of hardware cannot catch up the computational amount of three dimensional wave simulations as well as the re-reflection problems into the computational area due to the wave generation at the outer boundary. To estimate the wave transformation numerically, the research using the free surface trace approach of VOF (Volume-of-Fluid) method and three dimensional CFD (Computational Fluid Dynamics) model based on the Navier-Stokes equation and turbulence model has been tried to overcome the existing accuracy problems of two dimensional wave simulation model including non-linearity, dispersion, wave breaking and wave overtopping.

In 2DH wave model, the re-reflection problems into the computational area due to the wave generation at the outer boundary have been extensively studied. Larsen and Dancy(1983) had developed the internal wave generation and sponge layer methods and the other researchers has improved the internal wave generation schems (Lee and Suh, 1998; Lee et al., 2001; Kim et al., 2007). Lin and Liu (1999) had proposed the internal wave generation method using the mass source term and VOF scheme on 2D RANS (Reynolds Averaged Navier-Stokes) equation and Choi and Yoon (2009) had improved the internal wave generation method using momentum source wave-maker applied to the RANS equation model in a CFD code, FLUENT. Unfortunately, the method by Lin and Liu did not treat the directional wave generation because the model has been devised on the basis of two dimensional vertical RANS equation. Although Choi and Yoon's research proposed the directional wave-maker scheme for three dimensional extension, it cannot be applied to the strong nonlinear wave due to its limitation of

linear wave theory. Also, the abovementioned methods use the mass/momentum source at the fixed volume and the accurate vertical velocity profiles can be achieved after the wave propagates a sufficient distance.

In this study, two approaches including the mass source wave-maker and the self-adaptive wave-maker using mass flux are carried out the numerical simulations with FLOW-3D code, which is widely used in the field of coastal engineering. To prevent the reflection from the boundaries, the sponge layer method using porous media proposed by Choi and Yoon (2009) was included. In FLOW-3D model, linear monochromatic wave-maker, non-linear Stokes' 2nd order and 5th order wave-makers are devised and the application limits of wave-makers are suggested in 3D CFD code.

THEORY

The internal wave generation methods in 3D CFD code can be classified with the way of use of source as follows; that is, mass source, momentum source and mass flux.

Internal wave generation using mass source

Lin and Liu (1999) introduced the mass source term into the continuity equation of two dimensional vertical RANS equation to produce the internal wave-maker.

$$\frac{\partial u_i}{\partial x_i} = S(x,z,t) \qquad in \quad \Omega \tag{1}$$

where S is the mass source term in area Ω, $(\partial u_i)/(\partial x_i)$ is the partial velocity derivative with respect to space. The source term and free surface displacement (η) are defined with the following relationship.

$$\int_0^t \int_\Omega S(x,z,t) d\Omega dt = 2\int_0^t C\eta(t) dt \tag{2}$$

where C is the phase velocity(celerity) of the target wave, Ω is defined as the rectangular area which has the volume (V) of unit width acting the mass source term. For a linear monochromatic wave, $\eta(t) = H \sin(\omega t)/2$, where H is the wave height and ω is the wave frequency. Substituting $\eta(t)$ into (2), the following source term is achieved.

$$S(t) = \frac{2CH}{V}\sin(\omega t) \tag{3}$$

The 2nd order Stokes' wave as a nonlinear wave can be applied to the deep water and the intermediate sea and the free surface displacement (η) and the source term are defined as the following equation.

$$\eta(t) = \frac{H}{2}\left(\frac{\pi}{2} - \omega t - p_s\right) + \frac{H^2 k}{16}\frac{\cosh kd(2+\cosh 2kd)}{\sinh^3 kd}\cos\left(2\left(\frac{\pi}{2} - \omega t - p_s\right)\right) \tag{4}$$

$$S(t) = \frac{CH}{V}\cos\left(\frac{\pi}{2} - \omega t - p_s\right) + \frac{CH^2 k}{8V}\frac{\cosh kd(2+\cosh 2kd)}{\sinh^3 kd}\cos\left(2\left(\frac{\pi}{2} - \omega t - p_s\right)\right) \tag{5}$$

where p_s is the phase shift constant. To maintain the numerical stability, the source function is defined to must start from zero with the phase shift constant as follows;

$$p_s = \sin^{-1}\left(\frac{-a_s + \sqrt{a_s^2 + 8b_s^2}}{4b_s}\right) \tag{6}$$

where a_s = H/2, $b_s = \frac{H^2 k \cosh kd(2+2\cosh kd)}{16\sinh^3 kd}$

Internal wave generation using mass flux

The source term using mass flux in FLOW-3D adopted the implicit particle-fluid coupling scheme and the approximate momentum equations of the flow velocity U and the paticle velocity u_p from the time step n to n+1 at control volume are as follows;

$$M_f U^{n+1} = M_f \tilde{U} + m_p \delta k (u_p^{n+1} - U^{n+1}) \tag{8}$$

$$m_p u_p^{n+1} = m_p \tilde{u}_p - m_p \delta k (u_p^{n+1} - U^{n+1}) \tag{9}$$

where M_f is the fluid mass in control volume, m_p is the particle mass, k is a drag coefficient representing the interaction of fluid and particle. The superscript n+1 refers to the new time level values that are being computed as the advancement from step n to n+1 with step size of dt. A tilde over the velocities in the first term on the right hand sides of the equations indicates an estimate for the n+1 level velocity that contains all forces except for particle drag. These tilde velocities are computed explicitly in terms of known, time level n quantities. If there is no drag resistance, the tilde and n+1 level quantity is identical, and no further computations are necessary.

To solve Eq.(8) and (9) simultaneously, if the term of u_p^{n+1} is to be expressed with U^{n+1} and $\omega = \delta k/(1.0 + \delta k)$ is applied, the expression of U^{n+1} can be rearranged as the following equation.

$$U^{n+1} = \frac{M_f \tilde{U} + m_p \omega \tilde{u}_p}{M_f + m_p \omega} \tag{10}$$

The coupling effect between fluid and particle can be expressed with ω and if $\omega = 0$, the eq. of (10) can be reduced as the followings, which means there is no coupling effect between fluid and particle.

$$U^{n+1} = \tilde{U} \tag{11}$$

The fluid-particle interactions using the above eq. (7) are calculated with the internal calculation routing in FLOW-3D and the internal generation using mass flux in FLOW-3D can be produced such that total mass flux which is the equivalent to the horizontal particle velocities times the area of wave paddle has to be assigned with each wave paddle according to time to generate the accurate wave profile.

For a linear monochromatic wave, the horizontal water particle velocity can be calculated with the following equation.

$$u = \frac{gka}{\omega}\frac{\cosh k(d+z)}{\cosh kd}\cos(kx-\omega t) \tag{12}$$

In case of the 2nd order Stokes' wave, the following equation of horizontal particle velocity can be applied with the linear dispersion relationship.

$$u = \frac{gka}{\omega}\frac{\cosh k(d+z)}{\cosh kd}\cos(kx-\omega t) + \frac{3}{4}\frac{a^2\omega k\cosh 2k(d+z)}{\sinh^4 kd}\cos 2(kx-\omega t) \tag{13}$$

The linear dispersion relationship is as follows;

$$\omega = gk\tanh kd \tag{14}$$

The Stokes' fifth-order velocity potential by Skjelbreia and Hendrickson (1961) can be written in a series form.

$$\Phi = \frac{c}{k}\sum_{n=1}^{5}\lambda_n \cosh nks\sin n\theta \tag{15}$$

Where, the non-dimensional coefficients, λ_n, are written as

$$\lambda_1 = \lambda A_{11} + \lambda^3 A_{13} + \lambda^5 A_{15}$$
$$\lambda_2 = \lambda^2 A_{22} + \lambda^4 A_{24}$$
$$\lambda_3 = \lambda^3 A_{33} + \lambda^5 A_{35}$$
$$\lambda_4 = \lambda^4 A_{44}$$
$$\lambda_5 = \lambda^5 A_{55}$$

The detailed expressions of the above-mentioned parameters are found in the original paper by Skjelbreia and Hendrickson (1961).

The wave height and celerity are given by

$$H = \frac{2}{k}[\lambda + B_{33}\lambda^3 + (B_{35}+B_{55})\lambda^5] \tag{16}$$

$$c^2 = c_0^2[1+\lambda^2 C_1 + \lambda^4 C_2] \tag{17}$$

in which λ is an unknown along with the wave number. The quantities of λ and k are determined from eqs. (16) and (17) through an iterative technique.

The solution for the surface elevation and horizontal velocity with the fifth-order accuracy are written with the similar series form.

$$k\eta = \sum_{n=1}^{5}\eta_n' \cos n(kx-\omega t) \tag{18}$$

$$\frac{u}{c} = \sum_{n=1}^{5} n\phi_n' \cosh n(k(d+z))\cos n(kx-\omega t) \tag{19}$$

Where, the non-dimensional coefficients, η_n', ϕ_n', are written as

$$\eta_i' = \lambda, \quad \eta_2' = \lambda^2 B_{22} + \lambda^4 B_{24}, \quad \eta_3' = \lambda^3 B_{33} + \lambda^5 B_{35}$$
$$\eta_4' = \lambda^4 B_{44}, \quad \eta_5' = \lambda^5 B_{55}$$
$$\phi_1' = \lambda A_{11} + \lambda^3 A_{13} + \lambda^5 A_{15}, \quad \phi_2' = \lambda^2 A_{22} + \lambda^4 A_{24}$$
$$\phi_3' = \lambda^3 A_{33} + \lambda^5 A_{35}, \quad \phi_4' = \lambda^4 A_{44}, \quad \phi_5' = \lambda^5 A_{55}$$

Self-adaptive internal wave generation using mass flux

Because the internal wave generation using mass flux utilizes the vertical profile of horizontal particle velocity directly, it is more effective to reduce the evanescent mode than the previous mentioned approaches and possible to produce the more accurate wave profile. However, if the wave paddles are installed up to a fixed still water level, the mass balance cannot be fulfilled according the free surface elevation of wave. That is, although the free surface is located at the wave trough, the sufficient mass flux can be generated because the wave paddle is higher than the wave trough. However, in the case that the free surface elevation is located at the wave crest, the generated mass flux is not enough to produce the accurate wave profile. To overcome these shortcomings it is required to adopt the self-adaptive generation scheme, which changes the length of wave maker according to the free surface displacement. Using the small amplitude theory the equivalent coefficient of vertical distribution of horizontal particle velocities at the crest and trough which is matched with the vertical distribution of horizontal particle velocities at the still water level can be calculated with the integration of vertical distribution of horizontal particle velocities.

$$\int_{-d}^{0} u dz = \alpha\int_{-d}^{-a} u_{trough} dz = \beta\int_{-d}^{a} u_{crest} dz \tag{20}$$

$$\int_{-d}^{0} u dz = \alpha\int_{-d}^{-a} u_{trough} dz = \beta\int_{-d}^{a} u_{crest} dz \tag{21}$$

$$\alpha = \frac{\sinh kd}{\sinh k(d-a)} \quad at \quad wave\ trough \tag{22}$$

$$\beta = \frac{\sinh kd}{\sinh k(d+a)} \quad at \quad wave\ crest \tag{23}$$

where a denotes the amplitude of wave, d means the water depth at the still water level and is zero at he still water level.

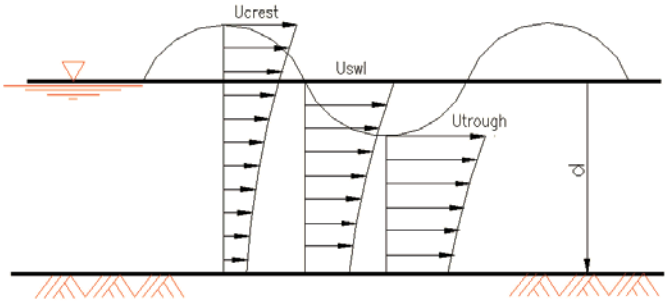

Fig. 1. Vertical distribution of horizontal particle velocities

If the above-mentioned coefficients α and β generalize to change to accommodate the change of free surface elevation, the coefficient of γ can be achieved with the following relation.

$$\gamma = \frac{\sinh kd}{\sinh k(d+\zeta)} \tag{24}$$

where ζ represents the free surface displacement. Though the self-adaptive internal wave generation using eq.(24) can consider the velocity profile on the basis of the change of the free surface elevation better than the methods of fixed wave paddle, the accuracy of numerical solution and the amounts of computations may depend on the size of spatial and time grids. Hence, the appropriate time step and grid spacing have to be determined through the several numerical

simulations under the incident wave conditions.

In case of Stoke' 2nd and 5th order wave theory, the similar coefficients are introduced with the more variables and the following equation is applied to the 2nd and 5th order wave theory according to the modes.

$$\delta = \frac{\sinh nkd}{\sinh nk(d+\zeta)}, \quad n = 1,2,...,5 \tag{25}$$

NUMERICAL SIMULATIONS

To make the internal wave generation of linear regular and nonlinear regular waves, the numerical simulations were carried out with the mass source internal wave generation proposed by Lin and Liu (1999) and mass flux internal wave generation proposed in this study. The numerical flume is constituted with the length of 21.6m and the height of 0.35m. The uniform grid is assigned with the size of 0.25m and the water depth is fixed as 20cm. The sponge layers of 3m were installed to remove the reflection effect at both side boundaries of numerical flume and the wave-maker was located at the center of flume. In case of mass source internal wave-maker, the unit volume of mass source composed of 1cm × 1cm × 1cm is located at the depth of 13cm ~ 14cm equivalent to one-third of water depth. Another case of mass flux internal wave-maker, the simulations were executed with four types of the length of wave paddle, which is changed from the sea bottom to the still water level, the wave crest, the wave trough and the free surface displacement, relatively.
Similar to Choi and Yoon (2009), the incident wave conditions are adopted with the wave period T=1 second, the amplitude a=0.5, 1.0, 1.5, 2.0cm and the comparisons between the result of free surface elevation and the analytical solution were carried out.

Internal wave generation using mass source

The results of numerical simulations of linear monochromatic wave using mass source are shown in the Fig.2 and those of Stokes' 2nd wave are shown in Fig.3.

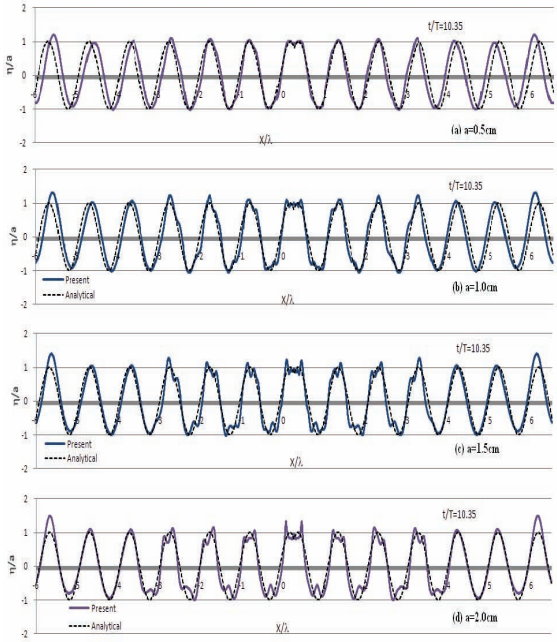

Fig. 2. Free Surface Displacement (a=0.5cm, 1.0cm, 1.5cm & 2.0cm) by Mass Source Method and Analytical Solution in Monochromatic Wave Internal Generation

As shown in Fig. 2., if the amplitude of monochromatic wave is smaller than 0.5cm, that is, the ratio of wave height and water depth (H/d=1/20) is one-twentieth, the generated wave profile is matched well with the sinusoidal analytic solution. However, in the case that the amplitude is increased, the wave profiles are distorted and the errors between the numerical solution and analytical solution are increased gradually. In case of Stokes' 2nd wave, if the amplitude of monochromatic wave is smaller than 1.0cm, that is, the ratio of wave height and water depth (H/d=1/10) is one-tenth, the simulated results detached from generation location show the good agreement with the analytical solutions. Similarly, the cases of larger amplitude show the distorted wave profile and the differences between the numerical solution and the analytical solution due to non-linear effect of waves.

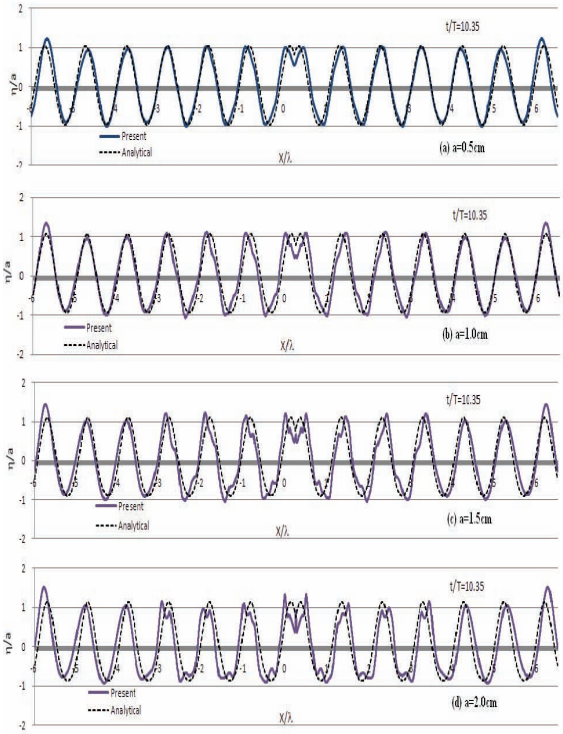

Fig. 3. Free Surface Displacement (a=0.5cm, 1.0cm, 1.5cm & 2.0cm) by Mass Source Method and Analytical Solution in Stokes' 2nd Order Wave Internal Generation

Internal wave generation using mass flux

The simulation results of linear monochromatic waves using mass flux wave-maker under the incident wave amplitude of 0.5cm are shown in Fig.4., which the case of four types of wave paddle that its length is changed from the sea bottom to the still water level, the wave crest, the wave trough and the free surface displacement. In the case of linear wave, the simulated wave profile is more accurate as well as the dispersion error is less compared to those of mass source method.

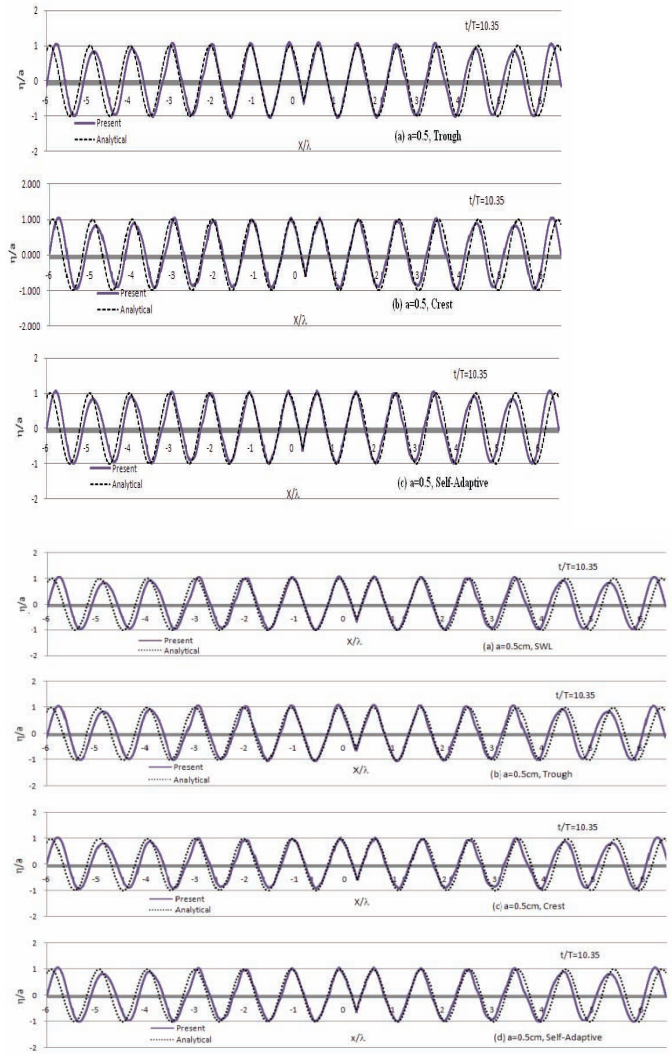

Fig. 4. Free Surface Displacements (a=0.5cm) on the basis of the various wave paddles by Mass Flux Method in Monochromatic Wave

The results of Stokes' 2nd wave internal generation at the still water level are shown in Fig.5 and Fig.6.. In case of Stokes' 2nd order wave, the distortion of wave profile is reduced and the accuracy of wave profile is improved considerably. Nevertheless, if the ratio of wave height and water depth is larger than one-tenth (H/d=1/10), the nonlinear effect of wave profile is appeared and the distortion and the error of results are increased more or less. Also, the method of internal wave generation using self-adaptive wave paddle shows the accurate results compared to the rest of generation methods.

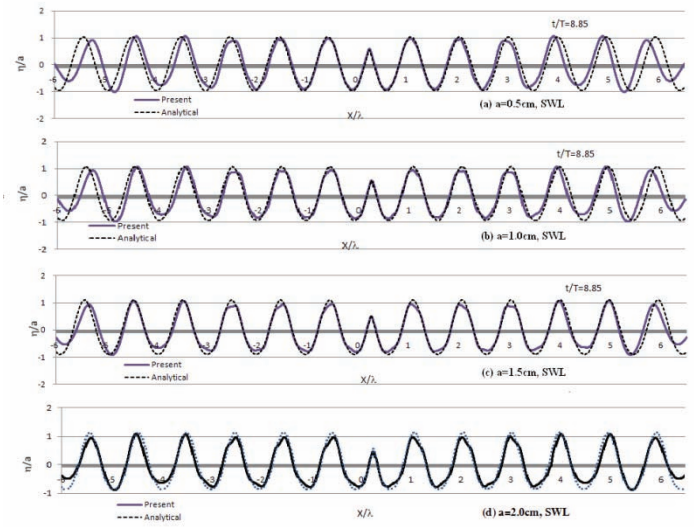

Fig. 5. Free Surface Displacement (a=0.5cm, 1.0cm, 1.5cm & 2.0cm) by Mass Flux Method in Stokes' 2nd Order Wave

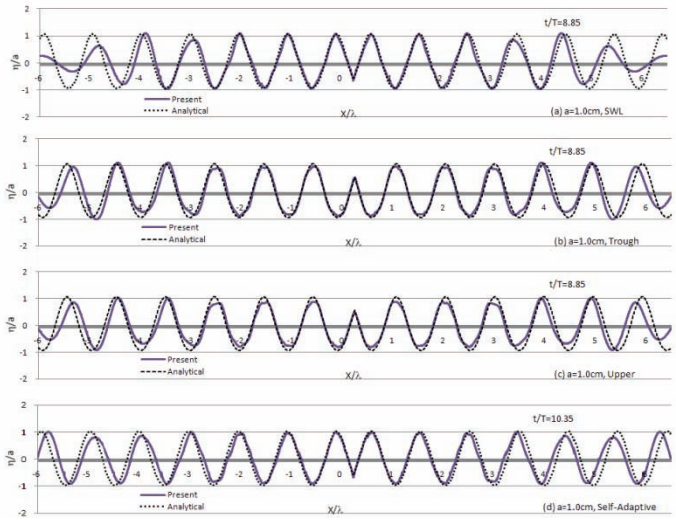

Fig. 6. Free Surface Displacement (a=1.0cm) on the basis of the various wave paddles by Mass Flux Method in Stokes' 2nd Order Wave

The results of Stokes' 5th wave internal generation at the still water level are shown in Fig.7 and Fig.8.. Compared to the previous methods, the wave generation using Stokes' 5th order wave gives the good agreement between the analytic solution and the simulated free surface elevation. But, similar to the previous results by the generation using Stokes' 2nd wave, the discrepancy between the theory and the numerical results grows as the ratio of wave height and water depth is larger than one-tenth (H/d=1/10). This tendency is the immediate cause by the nonlinear effect of the incident wave and the distortion and the error of results are increased more or less.

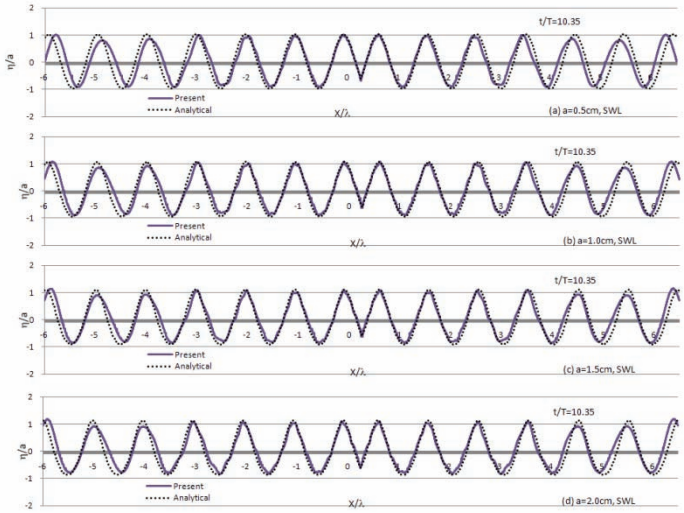

Fig. 7. Free Surface Displacement (a=1.0cm) on the basis of the various wave paddles by Mass Flux Method in Stokes' 5th Order Wave

In the Fig. 8, the numerical results using the different wave paddles are compared. Although the clear distinction between the various methods cannot be shown in Fig.8., it is clear that the better results can be achieved than those of linear waves or Stokes' 2nd wave.

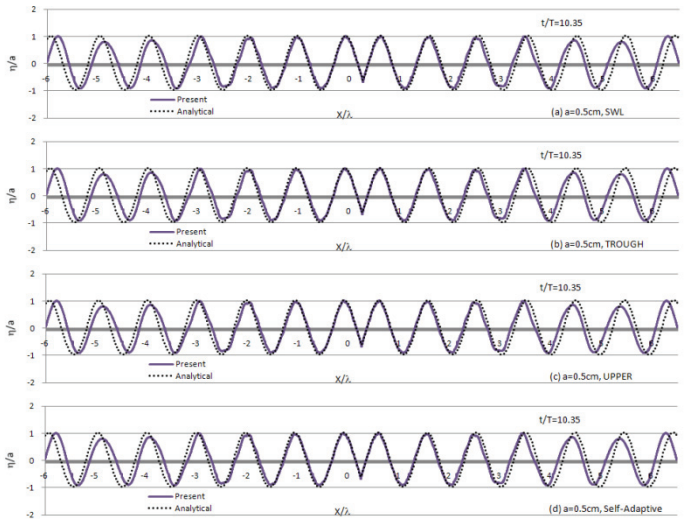

Fig. 8. Free Surface Displacement (a=0.5cm) on the basis of the various wave paddles by Mass Flux Method in Stokes' 5th Order Wave

CONCLUSIONS

In this study the internal wave generation using mass source proposed by Lin and Liu (1999) was applied in FLOW-3D CFD code and the internal wave generation method using the mass flux is devised. Compared to the existing wave-maker scheme, it is possible that the generated wave profiles of the internal wave-maker using mass flux show the better agreement with the analytical solutions even though the nonlinearity of wave was increased.

In case of the wave generation using linear wave theory, if the ratio of wave height and water depth is not larger than one-twentieth(H/d=1/20), the generated wave profile is matched well with the sinusoidal analytic solution. Because the wave profiles are distorted and the errors are increased gradually as long as the amplitude is increased, the internal wave-maker using mass flux using the nonlinear wave theory showed the better agreement with the analytical solutions. The mass flux internal wave-maker method applied in this study is expected to apply widely in the field of three dimensional wave problem study which is difficult to solve with two dimensional wave model and the high-order wave theory will be applied to expand the application limit of 3D wave-maker scheme from the offshore to the shallow water.

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