

## Drift Model for Two-Component Flows

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### Overview

In fluids composed of multiple components, e.g., fluid/particles, fluid/bubbles, fluid/fluid mixtures, where the components have different densities, it is observed that the components can assume different flow velocities. Velocity differences arise because the density differences result in non-uniform body forces. Often the differences in velocities can be very pronounced, for example, large raindrops falling through air or gravel sinking in water. Under many conditions, however, the relative velocities are small enough to be described as a “drift” of one component through the other. Examples are dust in air and silt in water.

The “drift” distinction has to do with whether or not the inertia of a dispersed component moving in a continuous component is significant. If the inertia of relative motion can be ignored, and the relative velocity reduced to a balance between a driving force (say gravity or a pressure gradient) and an opposing drag force between the components, then we can speak of a “drift-flux” approximation. Drift velocities are primarily responsible for the transport of mass and energy. Some momentum may be transported as well, but this is usually quite small and has been neglected in the *FLOW-3D<sup>I</sup>* drift model. A more complete analysis of when the “drift” assumption is valid can be found in the Flow Science, Inc. Technical Note, [FSI-08-TN77](#).

The concept behind the drift model is that the relative motion between components can be described as a continuum, rather than by discrete elements (e.g., particles). This enhances computational efficiency, as the need for tracking the motion and interaction of discrete elements do not need to be computed.

There are four different physical situations in which a drift velocity can be employed in *FLOW-3D*:

1. One fluid, variable density flows, where the fluid is a mixture of two components having densities  $\rho_1$  and  $\rho_2$ .
2. One fluid with solidification, where the fluid is a mixture of liquid and solid having densities  $\rho_1$  and  $\rho_2$ .
3. Two incompressible fluids having densities  $\rho_1$  and  $\rho_2$ .

4. A compressible gas with an incompressible component. In this case the density of the compressible gas is given by an equation-of-state, while the incompressible material has density  $\rho_1$ , which is always assumed to be much larger than the gas density.

### Drift Model Formulation

The formulation of the relative velocity in the drift approximation proceeds as follows. Suppose a flow is composed of two discrete components or phases, one the *continuous* phase, and the other the *dispersed* phase, which is discontinuous and surrounded by the continuous phase. Note that the same system of fluids can have opposite configurations: for the case of a small quantity of water contaminating a large volume of diesel fuel, the water is the dispersed phase; conversely, when small amounts of diesel fuel are present in water, the diesel fuel is the dispersed phase.

The following formulation uses subscripts 1 and 2 to denote the two components of the mixture. The selection as to which of the components is continuous and which is dispersed will be discussed later.

A mixture velocity is defined as a volume-weighted average  $\mathbf{u} = f_1 \mathbf{u}_1 + f_2 \mathbf{u}_2$ , such that  $\nabla \cdot \mathbf{u} = 0$  for the incompressible flow of a two-phase (or two-component) mixture. The volume fractions of the two components making up the mixture are denoted by  $f_1$  and  $f_2$ , such that  $f_1 + f_2 = 1$ .

If we presume that the two phases are incompressible, the momentum balance for phase 1 is

$$\frac{\partial \mathbf{u}_1}{\partial t} + \mathbf{u}_1 \cdot \nabla \mathbf{u}_1 = -\frac{1}{\rho_1} \nabla P + \mathbf{g} + \frac{K}{f_1 \rho_1} \mathbf{u}_r \quad (1)$$

while for phase 2 it is

$$\frac{\partial \mathbf{u}_2}{\partial t} + \mathbf{u}_2 \cdot \nabla \mathbf{u}_2 = -\frac{1}{\rho_2} \nabla P + \mathbf{g} - \frac{K}{f_2 \rho_2} \mathbf{u}_r \quad (2)$$

where  $\mathbf{u}_1$  and  $\mathbf{u}_2$  represent the *microscopic velocities* of the phases, respectively.  $f$  is the volume fraction phase 1. The microscopic velocity refers to the velocity of each phase about a small but finite volume of fluid.  $\mathbf{g}$  is the body force, and  $K$  is a drag coefficient that relates the interaction of the two phases.  $\mathbf{u}_r$  is the relative velocity difference between the phases:

$$\mathbf{u}_r = \mathbf{u}_2 - \mathbf{u}_1 \quad (3)$$

In terms of mixture and relative velocities, the component velocities are given by

$$\mathbf{u}_1 = \mathbf{u} - f_2 \mathbf{u}_r, \quad \mathbf{u}_2 = \mathbf{u} + f_1 \mathbf{u}_r \quad (4)$$

The goal of the drift-flux model is to compute the motion of the two phases relative to the volume-averaged velocity,  $\mathbf{u}$ . The volume-weighted average velocity is chosen rather than the mass-weighted average velocity because mass continuity of the mixture is automatically enforced, without alteration, i.e.,

$$\nabla \cdot \bar{\mathbf{u}} = 0 \quad (5)$$

and the mixture conservation of momentum is

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathbf{u}\mathbf{u} + \nabla \cdot (f_1 f_2 \mathbf{u}_r \mathbf{u}_r) = -\frac{1}{\rho} \nabla p + \mathbf{g} + \mathbf{F} \quad (6)$$

The term  $\mathbf{F}$ , defined as

$$\mathbf{F} = \frac{1}{\rho} \nabla p - \left( \frac{f_1}{\rho_1} + \frac{f_2}{\rho_2} \right) \nabla p + K \mathbf{u}_r \left( \frac{1}{\rho_1} - \frac{1}{\rho_2} \right),$$

is zero if the drift assumption given in Eq.8 below is used to evaluate the relative velocity. The vanishing of this term using the drift assumption is a big and convenient simplification since the mixture momentum and conservation equations, with the exception of the advection term involving the relative velocity in Eq.6, are then identical to the equations for a single incompressible fluid with the mixture density  $\rho = f_1 \rho_1 + f_2 \rho_2$ .

Subtracting Eq.1 from Eq. 2 produces an equation for the relative velocity where  $K$  is the drag coefficient per unit volume,

$$\frac{\partial \mathbf{u}_r}{\partial t} + \mathbf{u}_2 \cdot \nabla \mathbf{u}_2 - \mathbf{u}_1 \cdot \nabla \mathbf{u}_1 = \left( \frac{1}{\rho_1} - \frac{1}{\rho_2} \right) \nabla P - \left( \frac{1}{f_2 \rho_2} + \frac{1}{f_1 \rho_1} \right) K \mathbf{u}_r. \quad (7)$$

The goal is to determine the relative velocity  $\mathbf{u}_r$ . If we employ Eq.7 in its entirety, this would constitute a two-velocity-field model for two-component flow. For simplicity, however, we have chosen to make the drift-flux approximation, i.e., we assume that the relative velocity is nearly steady and the advection terms are negligible. [Flow Science Technical Note 77](#) investigates under what conditions these assumptions are valid.

However, using these assumptions, we have

$$\left( \frac{1}{\rho_1} - \frac{1}{\rho_2} \right) \nabla P = \left( \frac{f_1 \rho_1 + f_2 \rho_2}{f_1 f_2 \rho_1 \rho_2} \right) K \mathbf{u}_r. \quad (8)$$

Since the relative velocity  $\mathbf{u}_r$  is based on the microscopic velocities of each phase, then the drag force must contain some information about the volume fraction of the dispersed phase. For example, a suspension with an infinitesimally small amount of the suspended phase will produce an infinitesimally small momentum exchange between the components.

If we presume that the suspended phase is composed of particles that are the same size, and there are  $n$  of them in a unit volume, then

$$\mathbf{u}_r = \left( \frac{V_p}{K_p} \right) \frac{f(\rho_2 - \rho_1)}{\bar{\rho}} \nabla P \quad (9)$$

where  $V_p = f_2/n$  is the volume of a particle and  $K_p$  is the drag coefficient for a single particle moving with velocity magnitude  $U = |\mathbf{u}_r|$  through the continuous fluid.

An approximate drag expression, the one used in **FLOW-3D**, is that for a single particle with a Reynolds number dependent form. If  $U$  is the magnitude of relative velocity of a single particle moving in a continuous fluid with density  $\rho_c$  and viscosity  $\mu_c$ , then

$$K_p = \frac{1}{2} A_p \rho_c \left( C_d U + \frac{12\mu_c}{\rho_c R_p} \right) \quad (10)$$

where  $C_d$  is a drag coefficient (input variable  $dfcd$ ),  $R_p$  is the average particle radius (input variable  $dfrad$ ) and  $A_p$  is the cross sectional area of the particle (usually assumed to be spherical).

Because of the appearance of  $U$  in Eq. 10 an analytic solution (binomial equation) is used to solve for the relative velocity  $U$ .

### Correction for Interactions between Dispersed Elements

When the volume fraction of dispersed material is not vanishingly small, the use of a single particle drag force for computing the momentum exchange between components is incorrect. The most frequently used correction to account for particle/particle interactions is an experimentally-determined relation referred to as the Richardson-Zaki correlation.

The Richardson-Zaki correlation is dependent on the particle's Reynolds number,  $Re=(2RU/\nu)$ , where  $R$  is the particle radius and  $U$  the magnitude of its relative (drift) velocity in continuum fluid having kinematic viscosity  $\nu$ . The correlation consists of replacing the drift velocity by multiplying the computed value,  $\mathbf{u}_r$ , by the dispersed component volume fraction, here denoted by  $(1-f)$ , raised to a power  $\zeta$ ,

$$\mathbf{u}_r^{eff} = \mathbf{u}_r (1-f)^\zeta$$

The exponent  $\zeta$  is the product of  $rzmlt$  (an input parameter with a default value of 1.0) times  $\zeta_0$ , that is,  $\zeta = rzmlt * \zeta_0$  where

$Re < 1.0$	$\zeta_0 = 4.35/Re^{0.03}$
$1.0 < Re < 500$	$\zeta_0 = 4.45/Re^{0.1}$
$500 < Re$	$\zeta_0 = 2.39$

The adjusted, or effective value is used for the computation of all drift fluxes. A value of  $rzmlt=0$  eliminates the correction.

### Determining which component is dispersed

The dispersed component is defined to be that component having a volume fraction less than a specified value  $fdmax$ . Generally, one would expect that the continuous phase would have a volume fraction equal or greater than 0.5, but there can be situations where the transition between continuous and dispersed phases occurs at a value different than 0.5. To accommodate such situations the  $fdmax$  value is an input parameter, which has a default value of 0.5.

## Drift flux of scalar quantities

The transport equations for scalar quantities associated with fluids 1 and 2 can be expressed in terms of mixture and drift velocities in the following way. For the volume fractions  $f_1$  and  $f_2$  the velocities  $\mathbf{u}_1$  and  $\mathbf{u}_2$  in the original equations are replaced by their expressions in terms of the mixture and relative velocities using Eqs.4,

$$\begin{aligned}\frac{\partial f_1}{\partial t} + \nabla \cdot (f_1 \mathbf{u}_1) &= \frac{\partial f_1}{\partial t} + \nabla \cdot (f_1 \mathbf{u}) - \nabla \cdot (f_1 f_2 \mathbf{u}_r) = 0 \\ \frac{\partial f_2}{\partial t} + \nabla \cdot (f_2 \mathbf{u}_2) &= \frac{\partial f_2}{\partial t} + \nabla \cdot (f_2 \mathbf{u}) + \nabla \cdot (f_1 f_2 \mathbf{u}_r) = 0\end{aligned}\tag{11}$$

Because  $f_1 + f_2 = 1$  the sum of these equations is simply the incompressibility expression Eq.5, i.e., the vanishing of the divergence of the mixture velocity.

A similar transformation can be done for other scalar quantities that are densities associated with each fluid component. For example, consider the mass densities  $\rho_1$  and  $\rho_2$ , which satisfy the equations,

$$\begin{aligned}\frac{\partial f_1 \rho_1}{\partial t} + \nabla \cdot (f_1 \rho_1 \mathbf{u}_1) &= \frac{\partial f_1 \rho_1}{\partial t} + \nabla \cdot (f_1 \rho_1 \mathbf{u}) - \nabla \cdot (f_1 f_2 \rho_1 \mathbf{u}_r) = 0 \\ \frac{\partial f_2 \rho_2}{\partial t} + \nabla \cdot (f_2 \rho_2 \mathbf{u}_2) &= \frac{\partial f_2 \rho_2}{\partial t} + \nabla \cdot (f_2 \rho_2 \mathbf{u}) + \nabla \cdot (f_1 f_2 \rho_2 \mathbf{u}_r) = 0\end{aligned}\tag{12}$$

If we define the mixture density as  $\rho = f_1 \rho_1 + f_2 \rho_2$  then the equation for the mixture density is obtained by adding Eqs.12,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) + \nabla \cdot ((\rho_2 - \rho_1) f_1 f_2 \mathbf{u}_r) = 0\tag{13}$$

From this expression we see the general form of the drift flux contribution to the advection of scalar quantities. The product of  $f_1 f_2$  always multiplies the drift velocity and the quantity advected is the difference between the scalar value in fluid 2 minus the scalar value in fluid 1.

## Using the Drift Model in *FLOW-3D*

The drift model can be applied in a variety of situations. To activate the model requires inputting a non-zero value for the flag variable *ifdft*.

### One-fluid problems (*nmat=1*) with or without sharp free surfaces

Both components are assumed incompressible, usually with the continuous component density being *rhof* and the dispersed component density *rhofs*. In this case, the dispersed phase could be solidified fluid 1, or if the variable density model is used (*ifrho>1*), the dispersed component could be any material (e.g., solid particles or gas bubbles) having

density  $\rho_{hofs}$ . This case is suitable, for example, for dispersed solid matter or gas bubbles in a liquid that has a free surface.

To determine the viscous drag, Eq. 10, it is necessary to know the viscosity of the continuous component, which can be either component depending on its volume fraction. To accommodate this, special input is used for the viscosity of each of the individual components,  $\mu_{d1}$  and  $\mu_{d2}$ , while  $\mu_{m1}$  is the viscosity of the mixture (i.e., the viscosity of the “fluid” in a one-fluid problem). In this way the mixture can have a viscosity that differs from its individual components. The values of  $\mu_{d1}$  and  $\mu_{d2}$  default to  $\mu_{m1}$  if not otherwise specified.

#### Mixtures of two incompressible fluids ( $nmat=2$ , $icmprs=0$ and $itb=0$ )

The component densities are the usual  $\rho_{h1}$  and  $\rho_{h2}$ . This case is suitable for dispersed gas in a liquid, mixtures of two liquids, dispersed solid in a liquid or gas, and so forth.

#### Mixtures of compressible fluid and incompressible fluid ( $nmat=2$ , $icmprs=1$ and $itb=0$ )

Fluid 1 is incompressible with density  $\rho_{h1}$ . Fluid 2 is compressible and has a variable density. The dispersed component, as previously mentioned, is always the component having a volume fraction less than input parameter  $f_{dmax}$ . This case is for two-phase (including boiling/condensing) flows.

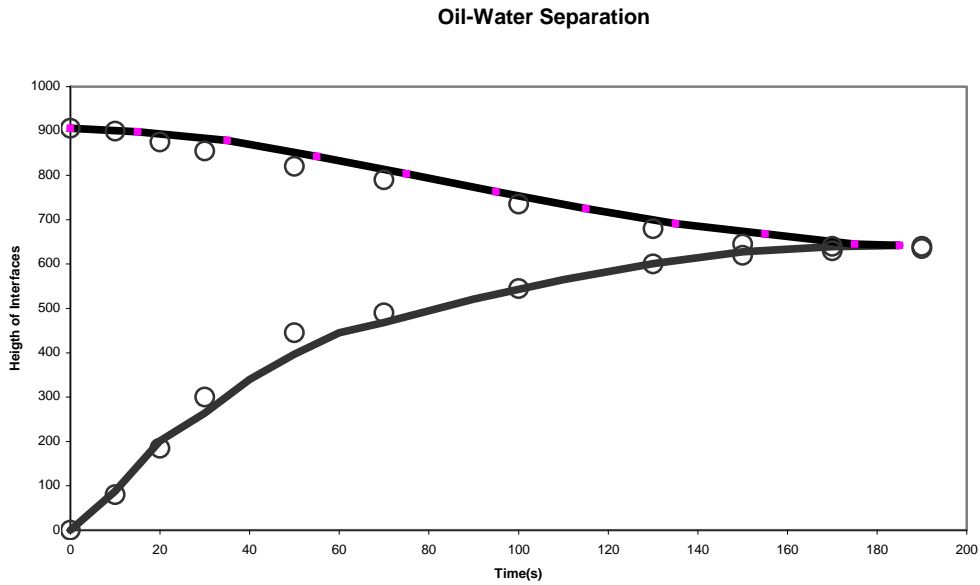
### **An Application Example—Separation of an Oil-Water Dispersion**

The separation of liquid-liquid dispersions is a problem encountered in many industrial situations. In the present case we consider a batch separator consisting of a vertical column containing a mixture of oil and water, as reported by Jeelani and Hartland, Ind. Eng. Chem. Res. 37, pp.547-554, 1998. Specifically the water is demineralized with density  $996\text{kg/m}^3$  and viscosity  $1\text{mPa}\cdot\text{s}$ , while the dispersed phase consists of decane in paraffin oil having a density  $837\text{kg/m}^3$  and viscosity  $1.26\text{mPa}\cdot\text{s}$ . The column height is  $0.915\text{m}$  and the initial volume fraction of oil is 30% ( $f_{oil}=0.3$ ) in the entire column.

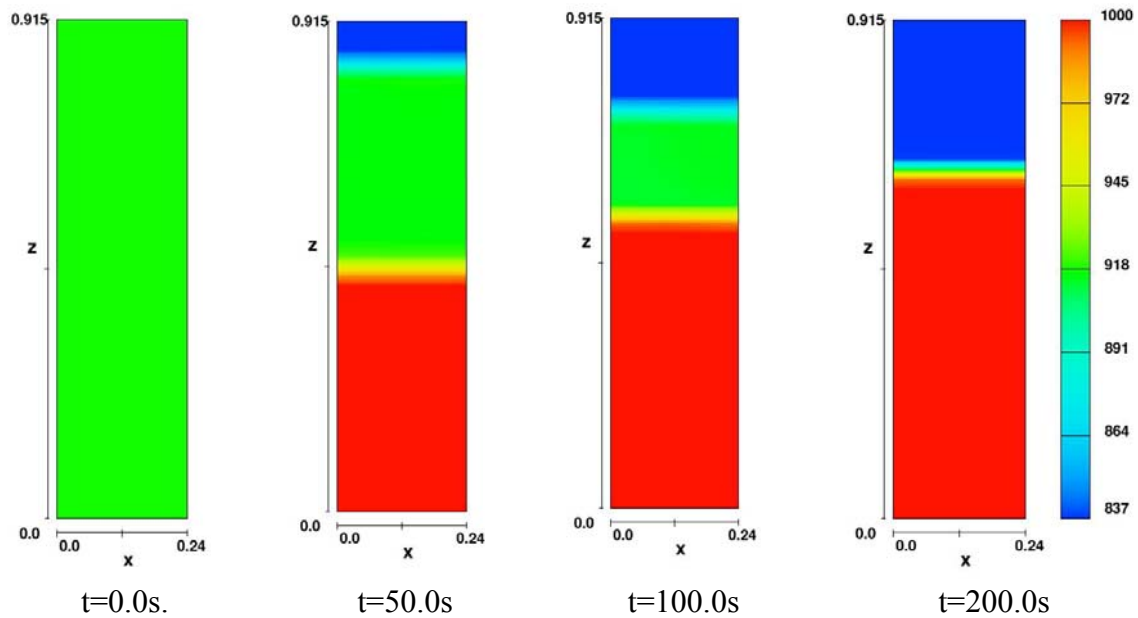
**FLOW-3D** models this situation as a two-fluid mixture in which the two phases are allowed to flow relative to one another using the drift flux model. There is no mixture velocity because the mixture in the column is in hydrostatic equilibrium. The dispersed phase is assumed to be in the form of droplets having a specified diameter. The relative, or drift, velocity of the droplets is computed from a balance of buoyancy, gravity and viscous forces acting on a drop as described in this Note. This velocity is also modified with a Richardson-Zaki correlation that accounts for droplet-droplet effects in terms of the dispersed volume fraction.

In Fig. 1 the bold lines are experimental data for the boundary heights of the top of the pure water region ( $H_s$ ) collecting at the bottom of the separator (bottom lines in plot) and the bottom of the pure oil region ( $H_c$ ) collecting at the top (top lines in the plot). In order for the oil drops to coalesce from a state of close packing to a state of pure oil it is necessary to switch the definition of the dispersed phase to be water instead of oil. This switch is done at the average volume fraction  $f_{oil}=0.5$ , so that the phase with the smallest

volume fraction is treated as the dispersed phase. Simulation results (open circles) are shown for a droplet diameter of 1.2mm. Figure 2 shows the evolution of the separation where blue is oil and red is water. Excellent agreement with the experimental data has been achieved at both early and late times.



**Figure 1.** Time history of separation of oil and water in a vertical column.



**Figure 2.** Evolution of the separation where blue is oil (top) and red is water (bottom). Color scale is density.

<sup>1</sup> *FLOW-3D* and TruVOF are registered trademarks of Flow Science, Inc in the USA and other countries.