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Modeling Sediment Scour

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Introduction

The sediment scour model predicts the behavior of packed and suspended sediment within the three-dimensional flow capabilities of ***FLOW-3D***®. Potential applications include erosion around bridge piers, weirs, dams and underwater pipelines, and removal and drifting of sand or snow over terrain. The model consists of two basic components: *drifting* and *lifting*. Drifting acts on sediment that is suspended in the flow; gravity (along with other body forces) causes the settling of the sediment. This model is based on the drift-flux model already incorporated into ***FLOW-3D***®. Lifting takes place only at the interface between the packed sediment and fluid and occurs where the local shear stress imposed by the liquid on the bed interface exceeds a critical value. The amount of lifting is proportional to the shear stress. In conjunction with the drifting and lifting models, a drag model is used to mimic the solid-like behavior of the sediment in regions where its concentration exceeds a cohesive solid fraction. The viscosity and density are functions of the sediment concentration; they are calculated as a function of the sediment concentration.

Model at a Glance

The sediment scour model uses two concentration fields: the suspended sediment and the packed sediment. The suspended sediment advects and drifts with the fluid due to the influence of the local pressure gradient. Suspended sediment originates from inflow boundaries or from erosion of packed sediment. The packed sediment, which does not advect, represents sediment that is bound by neighboring sediment particles. Its value divided by the critical packing concentration (SCRFCR in the input file) is the volume fraction of the cell that is occupied by packed sediment. Packed sediment can only move if it becomes eroded into suspended sediment at the packed sediment – fluid interface. Suspended sediment can become packed sediment if the fluid conditions are such that the sediment drifts towards the packed bed more quickly than it is eroded away. In regions where there exists packed sediment, fluid flow ceases; the drag is assumed to be infinite in such regions (*i.e.* the permeability is zero).

The sediment concentrations are stored as units of mass/volume (*e.g.* g/cm^3 in CGS). The solid volume fraction, f_S, is a measure of the fraction of the total volume that is occupied by the sediment; its value is derived from the sum of the two sediment concentrations divided by the microscopic sediment density, ρ_S (SCRRHO in the input file). The liquid fraction, f_L, is a measure of the fraction of the total volume occupied by liquid; it is equal to $1-f_S$. The solid fraction f_S is used for the viscosity and drag models.

The mean fluid viscosity is enhanced by the suspended sediment. This enhancement rises with sediment concentration until the solid volume fraction reaches the cohesive solid fraction, $f_{S,CO}$. Further rises in sediment concentration beyond solid volume fraction $f_{S,CO}$ do not cause the viscosity to rise, rather, the particles begin to interact with one another to cause solid-like behavior. This solid-like behavior is predicted by imposing a linear drag term to the momentum equation, as in flow through porous media. The viscosity is

$$\mu^{*} = \mu_f \left(1 - \frac{\min(f_S, f_{S,CO})}{f_{S,CR}}\right)^{-1.55} \tag{1}$$

where μ_f is the molecular viscosity of the liquid, μ^{*} is the average viscosity of the mixture and $f_{S,CR}$ is the critical solid fraction. Turbulence models used in ***FLOW-3D***® may also further raise the total viscosity. The critical solid fraction is the solid fraction at which the sediment particles are completely bound together in a solid-like mass; the drag coefficient K is infinite and fluid flow ceases. The drag coefficient is calculated as a function of the solid volume fraction and is

$$K = \begin{cases} 0 & if\ f_S < f_{S,CO} \\ \left[\dfrac{f_{S,CR} - f_{S,CO}}{f_{S,CR} - f_S}\right]\left[\dfrac{f_{S,CR} - f_{S,CO}}{f_{S,CR} - f_S} - 1\right] & if\ f_{S,CO} < f_S < f_{S,CR} \\ \infty & if\ f_S > f_{S,CR} \end{cases} . \tag{2}$$

It is incorporated into the momentum equation:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \bullet \nabla \mathbf{u} = -\nabla P + \nabla \bullet \boldsymbol{\tau} + \mathbf{g} - K\mathbf{u} . \tag{3}$$

The macroscopic density $\overline{\rho}$ is assumed to be a linear function of the sediment volume fraction:

$$\overline{\rho} = \rho_L + f_S(\rho_S - \rho_L) \tag{4}$$

where ρ_S and ρ_L are the microscopic densities of the sediment particles and the liquid, respectively.

Drift is the settling of the sediment particles due to the buoyant force on the particle. The drift velocity is

$$\mathbf{u}_{drift} = \frac{f_L d^2}{18\mu} \frac{\nabla P}{\overline{\rho}} (\rho_s - \rho_l) \tag{5}$$

where d is the mean sediment particle diameter, μ is the liquid viscosity and $\nabla P / \overline{\rho}$ is the mechanical potential gradient, or body acceleration. $\nabla P / \overline{\rho}$ is limited to 10 times the magnitude of gravity to eliminate the effects of possible numerical oscillation in pressure. In the vicinity of the liquid free surface, $\nabla P / \overline{\rho}$ is replaced by $\mathbf{g}$. f_L is included in Eq. 5 because drifting is limited by the presence of solid; in regions that are full of solid (f_L=0), $\mathbf{u}_{drift}$ falls to zero.

At the surface of the packed bed of sediment, the fluid shear stress acts to remove sediment; the amount eroded from the surface is a function of the fluid shear stress, the critical shear stress and the fluid and solid densities. A parameter familiar to hydraulic engineers is the *critical Shields parameter*[1]. This parameter correlates the minimum shear stress required to lift a sediment particle away from the packed bed interface for various particle diameters and densities and is commonly found in the literature. It is

$$\Theta_{crit} = \frac{\tau_{crit}}{g\left(\rho_f - \rho_s\right)d} \tag{6}$$

where τ_{crit} is the minimum shear stress along the bed surface needed to cause lifting of the sediment particles. This model is a *suspended-bed* model; it presumes that most of the sediment transport is by suspension and advection, where the sediment particles travel in the fluid away from the packed sediment interface. The sediment concentration in the liquid at the interface is predicted by other researchers[2,3,4]; the concentration in these models is a function of the 1.5th power of the *excess stress*, $(\tau - \tau_{crit})$. Another approach (*bed-load models*) is to calculate the flux of sediment moving along, or rolling over, the bed surface[5,6]; the flux is zero where the bed shear stress is less than the critical stress, and is dependent on the normalized bed stress where it is greater than the critical stress.

The goal of the model being developed here is to predict the local flux of sediment being eroded everywhere on a packed bed interface. A characteristic measure of velocity in the boundary layer is the *shear velocity*, $\sqrt{\tau/\rho}$. All of the empirical models in the literature contain a critical shear stress below which no erosion can occur. Therefore, one estimate of the rate at which sediment is lifted away from the packed bed interface is the *excess shear velocity*, $\sqrt{\left(\tau - \tau_{crit}\right)/\rho}$. The resulting formula for scour lift is

$$\mathbf{u}_{lift} = \alpha \mathbf{n}_{\mathrm{s}} \sqrt{\frac{\tau - \tau_{crit}}{\bar{\rho}}} \tag{7}$$

where $\mathbf{n}_\mathbf{s}$ is the vector normal to the packed bed surface. α (SCRALP in the input file) is a dimensionless parameter that represents the probability that a particle is lifted away from the packed surface; its value is typically 1 or smaller.

The angle of repose controls how steep a slope can be supported by the packed sediment in a quiescent flow region. A high angle of repose allows for a steep slope (e.g. clay), while sediment with a low angle of repose slides downward much more easily (e.g. sand). This is evident when one tries to create a pile of sand or dirt; the angle of repose is the angle the pile naturally makes relative to the horizontal. In the ***FLOW-3D***® sediment scour model, the angle of repose ζ is a parameter (SCRANG in the input file) entered in degrees; the actual angle φ is computed from the dot product of the packed interface normal with the gravity vector:

$$\varphi = \frac{\mathbf{n}_{\mathbf{interface}} \cdot \mathbf{g}}{\left|\mathbf{g}\right|} \tag{8}$$

where $\mathbf{n}_{\mathbf{interface}}$ is the surface normal vector and $\mathbf{g}$ is the gravity vector. The model works by altering the critical shear stress τ_{crit} at sloping interfaces. Where φ is zero (i.e. a horizontal surface with respect to gravity), the effective critical shear stress is equal to the critical shear stress τ^0_{crit}. On sloping interfaces, the critical shear stress is

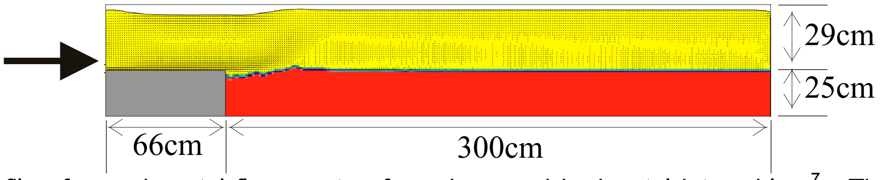


Figure 1. Profile of experimental flume setup for submerged horizontal jet problem[7]. The large arrow indicates the inflow of water through a narrow gap. The gap and pressure head were varied.

$$\tau_{crit} = \tau_{crit}^{0}\sqrt{1-\frac{\sin^{2}\varphi}{\sin^{2}\zeta}}\,. \tag{9}$$

Notice that when the local interface slope φ is equal to the angle of repose ζ, τ_{crit} is zero; sediment erodes from the interface due to any shearing action. Additionally, the model predicts negative values of τ_{crit} where φ is greater than ζ; in such regions sediment spontaneously erodes even in the absence of shearing action of the fluid.

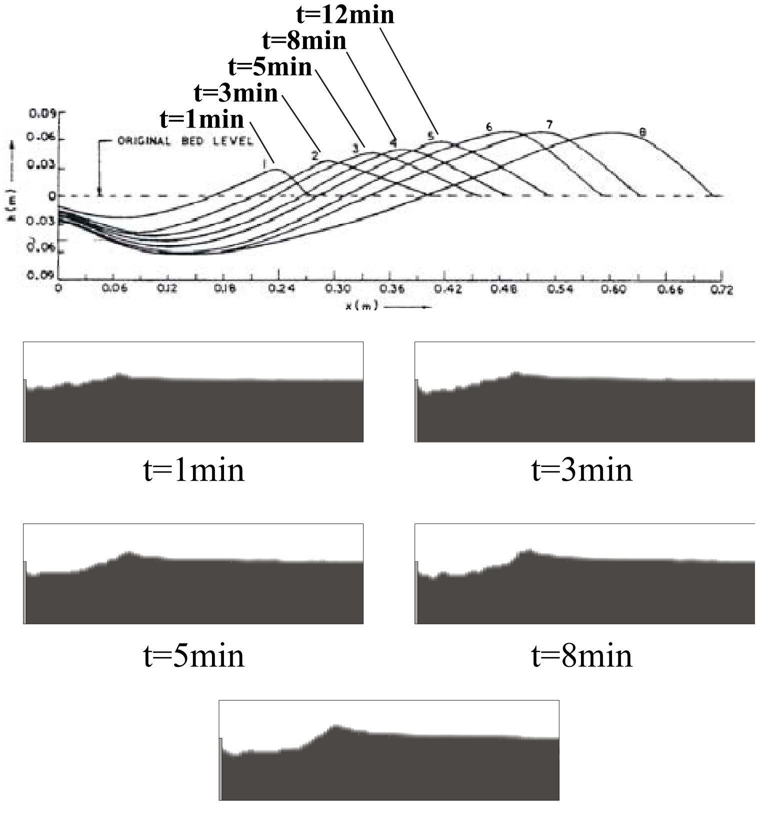
The motion of the suspended sediment in the system is described by the advection-diffusion equation, with the addition of the effect of drifting and lifting of the sediment:

$$\left(\frac{\partial c_s}{\partial t}\right)_{\mathbf{x}} + \mathbf{u}\cdot\nabla c_s = D\nabla^2 c_s - \mathbf{u}_{\mathbf{lift}}\cdot\nabla c_s - \mathbf{u}_{\mathbf{drift}}\cdot\nabla c_s\,. \tag{10}$$

Here $\mathbf{u}$ is the local fluid velocity, and $\mathbf{u}_{\mathbf{lift}}$ and $\mathbf{u}_{\mathbf{drift}}$ are the local lifting and drifting velocities, respectively. $\mathbf{u}_{\mathbf{lift}}$ is zero everywhere except in the vicinity of packed sediment interfaces where the local shear stress exceeds τ_{crit}. D is a diffusion coefficient; in ***FLOW-3D***® this is set by specifying the molecular diffusion coefficient CMSC in the input file, or the turbulence diffusion coefficient multiplier RMSC.

Test problem – submerged horizontal jet

Figure 1 shows the profile of the experimental flume setup for a submerged horizontal jet[7]. Water enters the flume through a narrow slit (from 2 to 5 cm in width) under pressure (2.6 to 20.7 cm of water). The resulting two-dimensional sheet of water flows over a solid apron, 66 cm in length, before contacting a packed bed of sand 300 cm in length and 25 cm deep. The channel is 60 cm wide, wide enough so that three-dimensional effects can be neglected near the middle of the flume. The scour profiles were measured down the centerline of the flume.



t=1min t=3min

t=5min t=8min

t=12min

Figure 2. Top plot shows the time evolution of the packed bed profile based on experiment[7]. The bottom plots show the predicted profiles from the ***FLOW-3D®*** simulation; the domain has been truncated in these plots so that only the edge of the apron and the first 134cm of the bed are visible.

Figure 2 shows the experimentally observed bed profile and the predicted results from the ***FLOW-3D®*** simulation. The shape and height of the predicted bed profiles compare well to the experimentally observed sand bed. Figure 3 is a plot of the depth of the deepest part of the scour hole and the height of the highest part of the deposited ridge; the experimental and predicted results are compared. The simulated height of the deposited ridge is predicted very closely to that of experiments, while the simulated scour hole is slightly over predicted compared to that of the experiments. Remember that the scour lift model is empirical and the α parameter represents a probability that a particle of sediment is lifted away from the packed bed interface; adjustment of this value may be needed to more accurately represent the lifting process at the packed bed interface. Figure 4 shows the comparisons for this and other experimental horizontal jet flume studies[7].

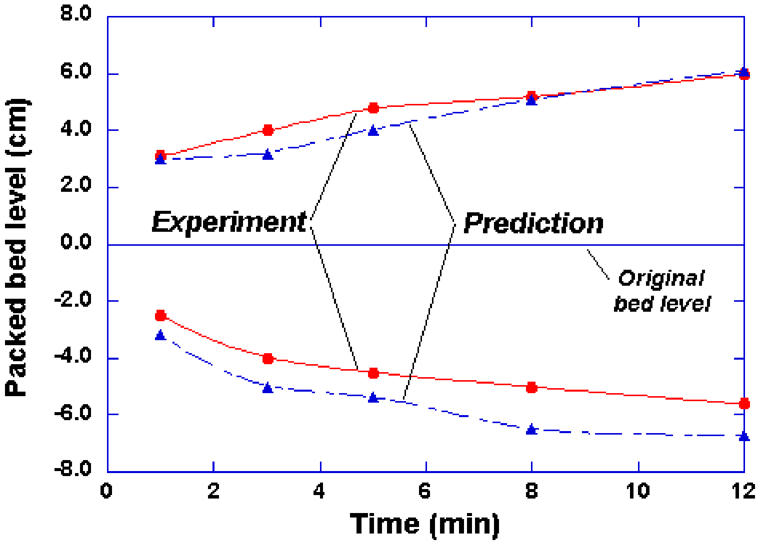


Figure 3. Plot of maximum scour hole depth and deposited ridge height as a function of time. Note that the discrepancy in the scour hole depth is due to the fact that the probability that a sediment particle will erode is less than 100%.

Summary

FLOW-3D$^{®}$'s sediment scour model is a straightforward approach to modeling erosion and deposition of sediment for fully three-dimensional flows. Predictions show that the equilibrium scour hole depth measurements compare well to experimental data, and rates of erosion can be extrapolated to different geometry and scales with the probability parameter, α. Limitations of the model are the particle size; large sediment particles cannot be accurately predicted because the assumptions of the drift model fail, and such systems typically require a bed load model, where the sediment particles roll or bounce over the surface of the packed bed interface, rather than becoming suspended in the fluid flow. Also, only one size and density of particle can be modeled; prediction of a range of particle sizes would require a scalar pair for each species and necessitate new lifting and packing models.

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