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THERMOCAPILLARY SAMPLE PROBLEMS

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It is well known that surface tension forces are primarily directed normal to a liquid surface and have a magnitude proportional to the local radius of curvature. What is not always appreciated is that a variation in the magnitude of surface tension generates and additional force that is tangential to the liquid surface. This force can exist even when the surface is flat.

Variations in surface tension may arise because of temperature variations or chemical reactions or nonuniform impruities introduced on the surface. Under the proper conditions, the tangential motion generated by a nonuniform surface tension may be a significant element in the dynamics.

Thermocapillary effects are important in the propagation of a flame over the surface of a liquid fuel, in weld pools, in crystal growth devices and in space applications. Because of its many application possibilities for thermocapillarity, a model for this phenomena has been added to ***FLOW-3D®***.

Unfortunately, there is very little experimental data available that can be used to validate this model. For this reason we have followed previous researchers and used an available, but approximate, analytic solution as the basis for validation. This comparision, as well as a situation having a simple force balance check, are described in this note.

Steady Two-Dimensional Slot Flow

Analytical solutions for thermocapillary flow in a slot have been given by Sen and Davis (A.K. Sen and S.H. Davis, "Steady Thermocapillary Flows in Two-Dimensional Slots," J. Fluid Mech. (1982), vol. 121, p.163). Their solution has been used by other model developers as a validation basis (e.g., G.P. Sasmal and J.I. Hochstein, "Marangoni Convection with a Curved and Deforming Free Surface in a Cavity," J. Fluids Eng. (1994), vol. 116, p.577).

The Sen and Davis results were obtained using a matched-expansion technique for flows in slots having a small height to length ratio. Some numerical work was necessary in their solutions to compute terms used in the expansion. They did not use a computational fluid dynamics technique.

The case chosen for comparison is a slot having a height to width ratio of $d/L=0.2$. The left side wall is heated and the right side wall is cooled, with a total temperature difference of ΔT. The bottom of the slot is insulated.

The problem can be characterized by non-dimensional parameters:

Aspect Ratio = d/L=0.2
Reynolds Number = $\gamma A\Delta Td\rho/\mu^2$= 1.0
Capillary Number = $\gamma A\Delta T/\sigma$= 0.008
Marangoni Number =$\gamma A\Delta Td/(\mu\kappa)$= 0.2
Prandtl Number =$\mu/(\rho\kappa)$= 0.2,

where γ is the negative derivative of surface tension with respect to temperature, ρ is the fluid density, μ is the fluid dynamic viscosity, σ is the coefficient of surface tension and κ is the fluid thermal diffusivity.

Sen and Davis studied several cases corresponding to different boundary conditions for the contact line where the fluid surface meets the sides of the slot. The case we have modeled is a floating contact line with 90 degree contact angle at each side wall. Sen and Davis also considered heat conduction between the liquid and air, but concluded that this effect had little effect on the results, so we have neglected this effect in our simulations.

The ***FLOW-3D***® computations were performed with a grid consisting of 40 cells across the width of the slot and 15 cells resolving the depth of fluid. Material properties were selected to give the non-dimensional parameter values indicated above.

Figure 1a-c shows the steady-state computed results. Actually, the results are not exactly steady as there is a small oscillatory component that appears not to be damped, but this unsteadiness is well within the accuracy of the approximate solution. We compute a flow in qualitative agreement with the analytic result, except that our velocity distribution, Fig. 1a, shows a tendency toward a double cell structure, while the analysis considered only a single cell flow pattern. It is interesting to note that Sen and Davis comment on a two-cell structure appearing in higher-order terms in the stream function.

Temperature contours, Fig. 1b, are nearly uniformly spaced across the slot, which is consistent with the assumptions made in the theoretical analysis of nearly pure conduction. Corrections to the linear distribution of temperature appear only at the second-order level in the expansion.

The most interesting feature to compare is the height increase of the free surface at the cold wall and height depression at the hot wall, Fig.1c. Numerical values are not given by Sen and Davis but estimates can be made from their plot (Fig. 9) of the solution. Using a ruler and scaling in terms of the initial depth of 0.2 units, the measures surface heights were 0.188 and 0.212. However, the analytical results are stated as being accurate only to order A^2=0.04, so we can't claim values more accurate than 0.19 and 0.21.

The surface heights computed by ***FLOW-3D***® were 0.192 and 0.207, which when rounded to the same level of accuracy, reduce exactly to the analytical results 0.19 and 0.21.

It should be mentioned that Sasmal and Hochstein in their paper state that surface heights obtained by Sen and Davis were 0.175 and 0.225. These values were then used as the benchmark for their computational method. We have been unable to find any such values quoted by Sen and Davis. They are certainly not the values in the example given by Sen and Davis. We also note that additional results obtained with ***FLOW-3D***® to compare with cases reported by Sasmal and Hochstein agree only qualitatively and not quantitatively with their results. In all cases our surface amplitudes are less than those reported by Sasmal and Hochstein.

Since we agree with the benchmark data, in so far as we were able to determine it from the Sen and Davis paper, we believe our results to be more reliable than those of Sasmal and Hochstein. This conclusion is supported by the next test.

Force Balance Test

As a further check on the ***FLOW-3D***® thermocapillary model we have performed a simple, non-dimensional, test involving a balance between the thermocapillary surface force and viscous shear stress.

A layer of fluid 1.0 unit deep having a horizontal free surface and 10 units wide is subjected to a uniform temperature gradient from left to right. At the bottom of the layer there is a no-slip solid wall. At the right side boundary we fix the fluid height at 1.0 unit with hydrostatic pressure and a temperature of 2.0 units (i.e., this is a specified pressure boundary condition). At the left side the same condition is used, except the temperature is set at 1.0 units.

The computational grid contains 10 cells vertically and 21 cells horizontally. The vertical grid is not exactly uniform because a grid line was forced at z=0.95. Fluid occupies the lower 6 cells plus 0.36 of the 7th cell. The derivative of surface tension with temperature is 0.01 and the dynamic viscosity is 1.0. The thermocapillary force is computed in terms of the temperature gradient, but since we specified temperatures in the centers of the left and right boundary cells the distance between the wall temperatures is greater than the width of the computed region by one additional mesh increment. Taking this into account the thermocapillary force per unit surface area is $\partial\sigma/\partial s=.01(1/10.476)=0.00095$, where σ is the surface tension and s is distance along the surface.

In this problem the surface remains flat so there are no normal surface tension forces. However, the magnitude of surface tension is higher at the left side (i.e., the cooler side) causing a thermocapillary force that drives flow to the left. This flow is resisted by a viscous shear stress. At steady conditions the computed velocity magnitude at the surface is 0.00096. The retarding shear stress per unit surface area is then $\mu\partial u/\partial z=1.0(0.00096/1.0)=0.00096$, which is within 1% of the thermocapillary force.

Summary

The good agreement obtained in the two cases described above lends support for the accuracy of the thermocapillary model in ***FLOW-3D***®. Of course, real experimental data would be more satisfying, but until such data is available we must be content with comparisons with analytic models.

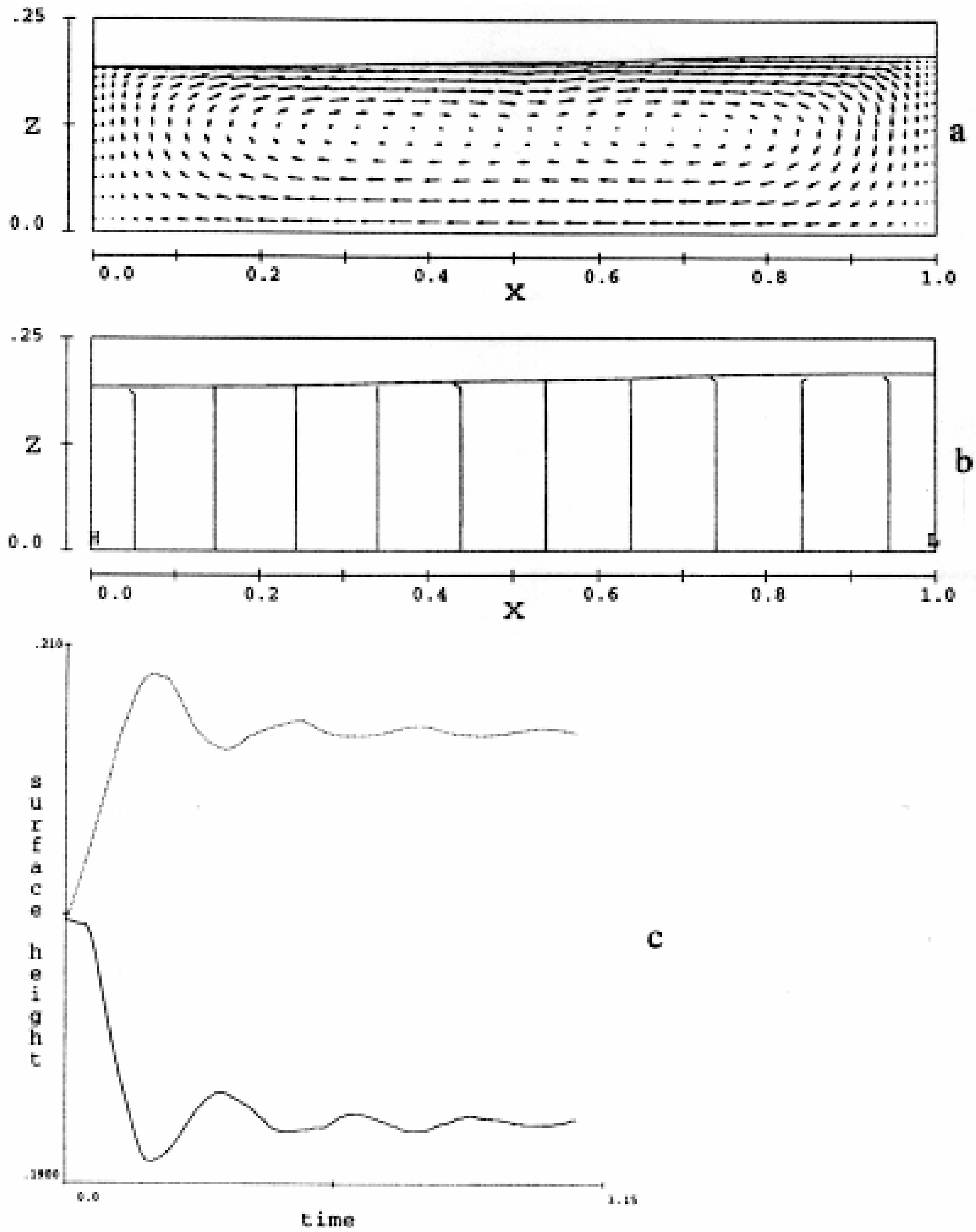

Figure 1. Computed results for Sen and Davis problem, (a) velocity field, (b) temperature contour, (c) time histories of fluid heights at side walls.