FSI-92-TN36

IDENTIFICATION AND TREATMENT OF STIFF BUBBLE PROBLEMS

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October 20, 1992

BACKGROUND

It has been observed in a number of instances that FLOW-3D can develop severe difficulties when high pressure, adiabatic, gas bubbles are present in a calculation. To help eliminate these difficulties we added to Version 5.0 of the code an implicit bubble model. The purpose of this addition was to anticipate bubble pressure changes computed at the end of a cycle and include them in the usual pressure-velocity iteration process.

The implicit bubble model works well if the "stiffness" of the bubbles is not too large. In the next section we shall define stiffness, but for now we mean by a stiff bubble one whose anticipated and actual pressure changes are very different. In other words, a stiff bubble is, by definition, one in which the end-of-cycle pressure is not well approximated by the implicit model. When stiff bubbles occur FLOW-3D has large pressure changes every cycle and corresponding large fluctuations in velocities and other quantities.

In this note we describe the computational problem of stiff bubbles in more detail so that users will be better able to identify its occurrence. We also offer some suggestions for ways to get around the stiffness problem.

STIFFNESS

Consider the problem of a two-dimensional gas bubble located in the center of a channel. Across the ends of the channel a pressure difference of magnitude dP is applied that establishes a flow in the channel with average velocity U. The mean applied pressure, P, is equal to the initial pressure in the bubble so that the flow should carry the bubble through the channel with little change in its volume or shape.

If we assign a ratio of specific heats, gamma (γ), to the bubble we can compute the change in pressure δP caused by a change in bubble volume δV. For this computation we assume the usual adiabatic relation,

$$PV^{\gamma} = \text{constant}.$$

The result is,

$$\delta P = -\gamma P(\delta V/V).$$

If the bubble is only resolved by a few computational cells, it is nearly impossible to estimate $\delta V/V$ accurately in the implicit bubble model because it uses a crude approximation for the advection of fluid fraction. This, then, will lead to a large change in bubble pressure. Even when the volume change is approximated well, however, a large mean pressure P can still result in a large change in pressure δP because P multiplies the volume change on the right side of the above equation.

Just how large a δP can one tolerate in a calculation? This is the essence of the stiffness problem. In the channel flow example, the applied pressure difference across the ends of the channel, dP, defines the allowable range of pressure variations for the calculation. Thus, we must demand that the magnitude of the change in volume of the bubble in one computational time cycle satisfy,

$$\delta V/V < (dP/P)/\gamma.$$

If instead of an applied pressure drop across the channel we had imposed a mean velocity U at one of the ends, a measure of allowable pressure change would be,

$$\delta P \approx \tfrac{1}{2}\rho U^2,$$

and the allowable volume change would be limited by,

$$\delta V/V < (\tfrac{1}{2}\rho U^2)/(\gamma P).$$

To see what these relations imply, suppose the bubble fluid is air (γ=1.4) at an atmospheric pressure of 1.0E+6 dynes/cm^2. Let the channel be subjected to a pressure difference of 1.0E+4 dynes/cm^2. According to the above relation the relative volume change in one cycle must be less than 0.007, that is, less than 1%. If this high accuracy is not achieved, volume changes in the bubble will produce pressure changes that exceed the applied pressure across the channel. Should this happen the entire calculation will rapidly become meaningless.

We now see what it means for an adiabatic bubble to be stiff; volume changes must be estimated very accurately in the implicit bubble model or else pressure perturbations will be generated that exceed the range of physically acceptable pressures. In the next section we discuss ways to overcome the stiffness problem.

OVERCOMING STIFFNESS

If stiffness, as described above, appears to be a problem in a FLOW-3D simulation and the implicit bubble model (IMPBUB=1) does not provide a solution then the user must resort to other measures. Three such measures are described here.

Attack Number One

For cases where the stiffness is not too excessive it may be sufficient to simply reduce the maximum time-step size or increase the grid resolution of the bubbles. Perhaps both actions should be taken for greater effect. What is needed is a reduction in the error of the estimated bubble volume by the implicit bubble scheme. More resolution and smaller time increments are two means of reducing this error.

Attack Number Two

For problems where stiffness is more than what can be handled by Attack Number One the user should try to reduce the stiffness artificially. A good way to do this is by lowering the mean pressure level (reference pressure) so that the allowable volume change for bubbles is increased.

For example, in the sample bubble problem described earlier, suppose we agree that bubble volume changes as large as 3.5% are acceptable, even though the original problem indicated a maximum volume change of only 0.7%. Increasing the allowable volume change by a factor of 5.0 corresponds to having a mean pressure 5.0 times lower, P=2.0E+5 dynes/cm^2 instead of 1.0E+6 dynes/cm^2. Except for interpretation, changing pressures by an additive constant usually has no other effect than reducing the stiffness (i.e., allowed bubble volume changes). This is often a good and simple means of overcoming stiffness.

Attack Number Three

In extreme cases of stiffness the bubble volume changes so little that an incompressible fluid assumption may be valid. This suggests the use of either the two-incompressible fluid model (NMAT=2) or a one-fluid, variable density model (NMAT=1, IFRHO=2 or 3). Unfortunately, there are some restrictions when these alternative models are used.

The two-incompressible fluid case cannot be used with free surfaces. That is, all regions void of fluid number 1 must be occupied with fluid number 2. If fluid is added or removed from the mesh at boundaries or through sources, there must be a compensating source or sink somewhere else in order for global incompressibility to be satisfied.

In the variable density model there can be a free surface, but heat transfer (and buoyancy) options cannot be invoked with this model.

In either of the above cases there is a further difficulty that may cause problems. If the fluid and bubble densities are very different (e.g., density ratios exceeding about 10) then the code may have difficulty achieving pressure convergence and may not accurately treat the density interface motion. This last problem is especially pronounced when there is a significant velocity slip across the interface, something that FLOW-3D does not properly take into account.

One situation where the two-fluid model can be used, for example, is in acoustic problems. Each fluid can be given different acoustic impedances (using RCSQL and RCSQV) and because material flow speeds are small with respect to sound speeds a poor interface-slip treatment is not important.

Code modifications are being worked on that will do a better job of two-fluid interfaces. If these advances work they will offer a better option for the treatment of very stiff bubbles.

Please call the Flow Science technical staff for up-to-date information on what to do if you have stiffness difficulties and none of these attacks can be made to work.

ILLUSTRATIVE EXAMPLES

The computational problem and the various solutions of the problem discussed above may be better understood by studying the results obtained from a simple example. Let us compute the motion of a two-dimensional bubble of radius 0.5 cm located at the center of a channel of width 3.0 cm and length 3.0 cm. Initially the fluid is at rest, but a pressure gradient is applied across the channel. Because of the applied pressure gradient, the bubble should move upward and should also flatten in the direction of motion.

For a first calculation let the initial pressure of the bubble be 1.0E+6 dynes/cm^2 and the applied pressure difference be 1.0E+4 dynes/cm^2. The initial bubble pressure is equal to the average fluid pressure in the channel. Computational results for this case without the implicit bubble model are given in Fig.1 at times 0.02 and 0.04 seconds. Pressure fluctuations develop that greatly exceed the pressure difference applied to the channel. Furthermore, the pressure fluctuations are growing with time and vary significantly above and below the initial pressure. In the t=0.04s plot the fluid velocities at the bottom of the bubble are directed down, indicating that the bubble is undergoing large oscillations instead of moving upward as expected. These are extremely poor results.

We selected a coarse mesh for this example (10x20) so that estimates of bubble volume changes by the implicit model would contain non-negligible errors. This is evident in Fig. 2 which shows a repeat of the first calculation using the implicit bubble model. The computed results are no better than those obtained without the implicit model.

For Fig.3 the time-step size was not allowed to exceed 1.0E-4 seconds. This is about 20 to 40 times smaller than the stability limit (assuming there are no big pressure variations). With the restricted time step the bubble volume changes in one time cycle are much smaller, and the calculation proceeds without difficulty. Here we see a bubble moving smoothly upward and flattening as expected. There are no large pressure fluctuations.

Finally, in Fig.4 we show an example where the mean pressure (at bubble and boundaries) was reduced by one order of magnitude, but the time-step size was not restricted. Except for pressures, the results are nearly identical to those in Fig.3 and, therefore, provide another solution to the stiffness problem.

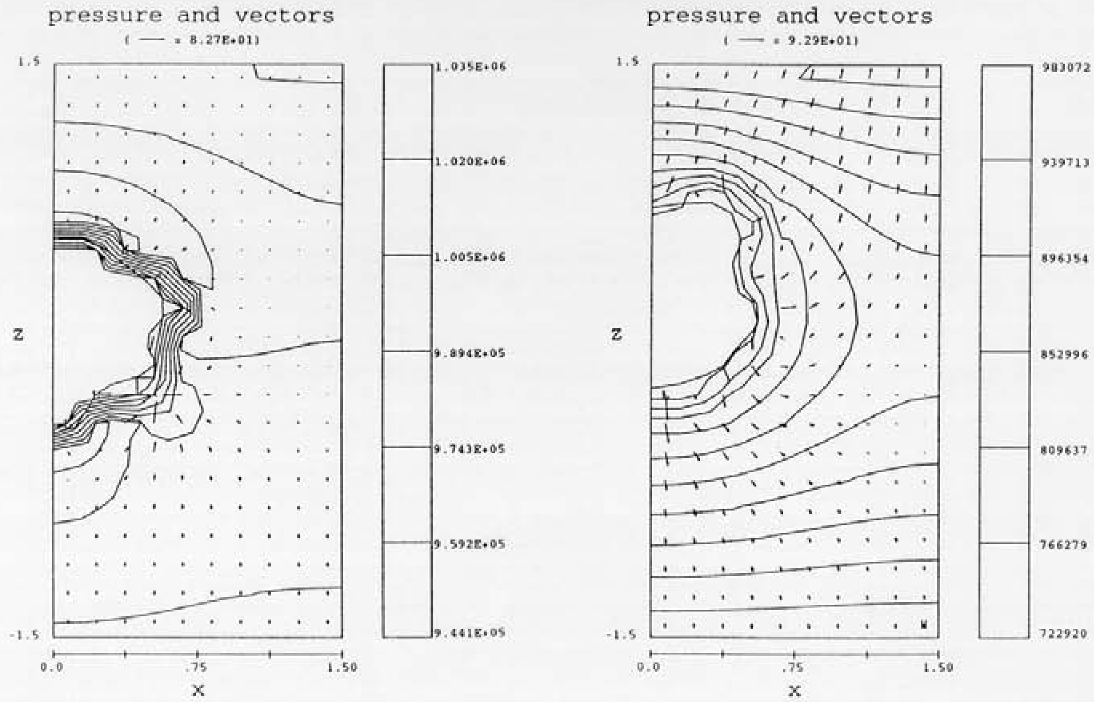

Fig. 1. Explicit bubble model for stiff bubble conditions. Velocities and pressures at 0.02 and 0.04 s.

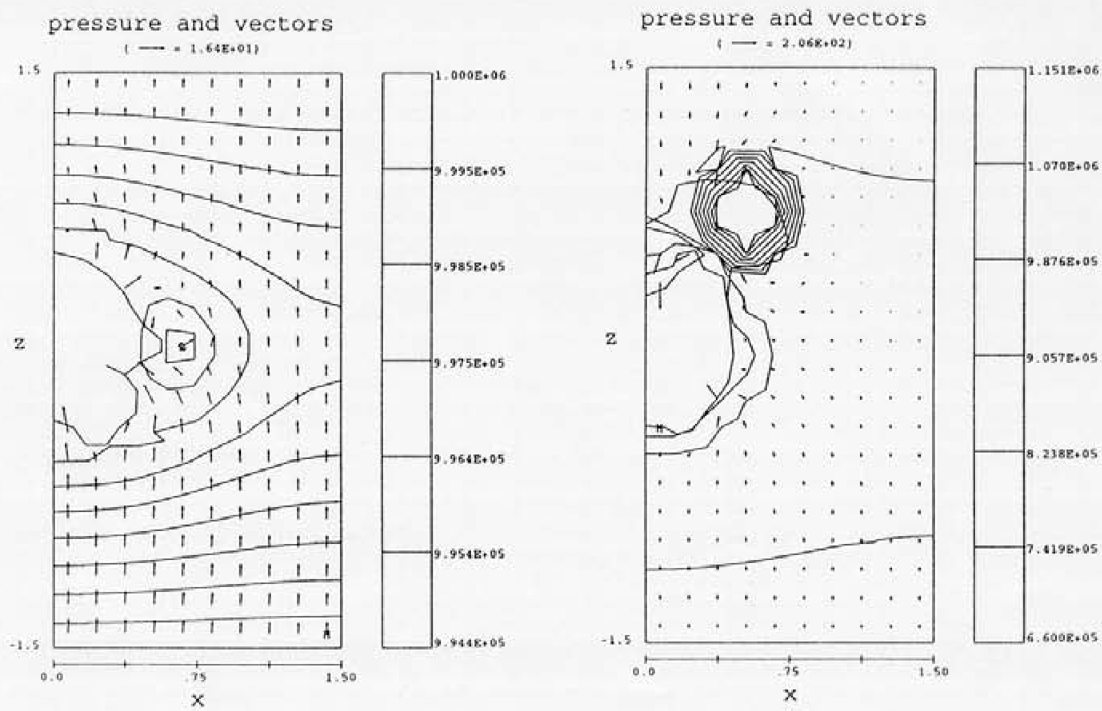

Fig. 2. Implicit bubble model for stiff bubble conditions. Velocities and pressures at 0.02 and 0.04 s.

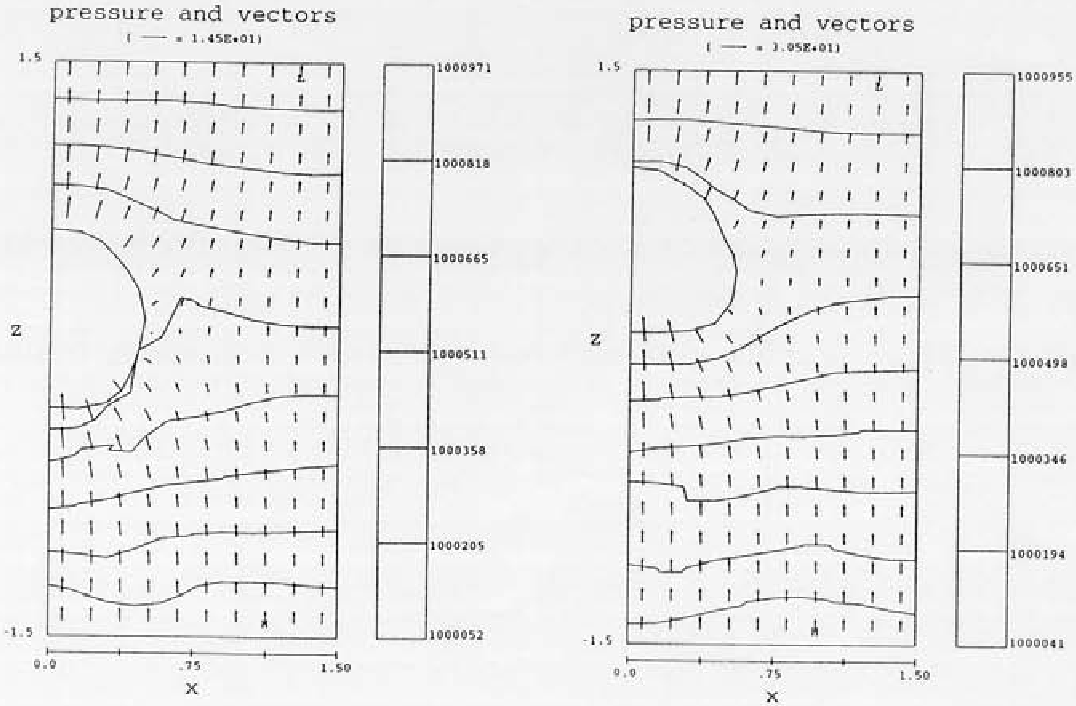

Fig. 3. Reduced δt results for stiff bubble conditions in Fig.1. Velocities and pressures at 0.02 and 0.04 s.

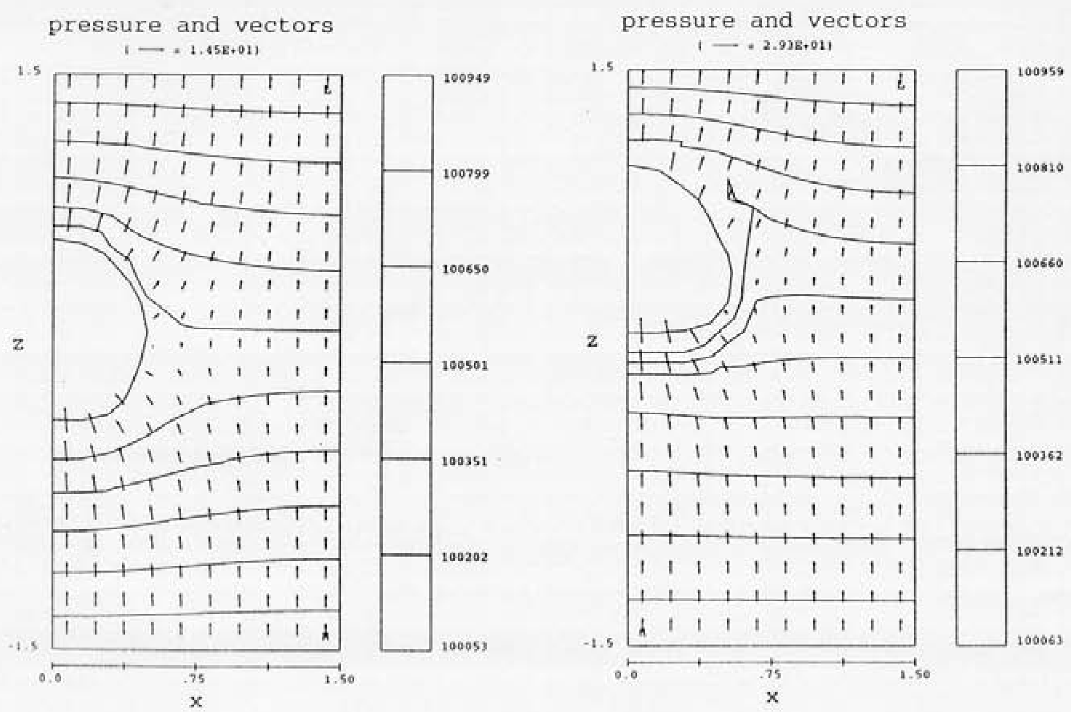

Fig. 4. Reduced pressure results for conditions in Fig.1. Velocities and pressures at 0.02 and 0.04 s.